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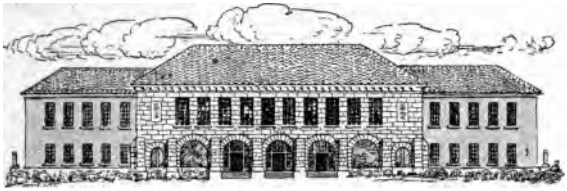

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CONCRETE GEOMETRY

FOR BEGINNERS

BY

A. R. HORNBROOK, A.M.

TEACHER OF MATHEMATICS IN HIGH SCHOOL, EVANSVILLE, IND.

NEW YORK ·:· CINCINNATI ·:· CHICAGO

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A. R. H. CONCRETE GEOM.
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PREFACE.

"If you will put that into figures for me, perhaps I can understand it," was said to the author of this book by a pupil, who, having tried in vain to grasp a general proposition in demonstrative geometry, sought a kind of assistance which he had found to be useful. In his demand for figures (by which he meant numbers) he was expressing the universal demand of the learning mind for the concrete and the particular, as stepping stones to the abstract and the general.

Much of the material of this book was prepared to meet this demand on the part of the author's own pupils during several years of teaching, and all of it has been subjected to the test of the schoolroom under plans which encourage on the part of the pupils the freest disclosures of the mental processes induced by the work.

The aim of the work is to awaken gradually, by simple and natural methods, the mathematical consciousness of the child and to guide his perceptions in such a way as to lead him to lay a firm foundation for demonstrative geometry by means of his own observations and inventions. The recognition of different geometric forms and of their numerical relations, and the practicing of the geometric imagination, constitute for children a useful and delightful exercise, which this book is intended to promote.

The author has selected from standard mathematical works the statements of some important facts and principles that are capable of concrete demonstration from the standpoint of the child, and has presented them in various ways and in different relations. Emphasis is placed upon the number relations of geometric forms, as a means of arousing those definite and exact ideas of form that are induced by actual measurements and computations.

As a step towards the correlation of studies which is acknowledged to promote economy in educational effort, the algebraic equation in its simplest forms and uses has been introduced, as

a convenient instrument for determining values in connection with geometric forms. Teachers who have used the algebraic equation in arithmetical or geometrical work are agreed that its use not only facilitates the work in hand, but that it tends to establish the habit of readily symbolizing the unknown, which is so valuable in mathematical work.

The author has tried, at the expense sometimes of completeness in arrangement, to avoid the pedagogical blunder of putting similars in proximity. Supplement and complement, rhomboid and rhombus, trapezoid and trapezium, are separated in presentation, with the hope that the first one presented will have made an ineffaceable impression of itself upon the mind of the pupil before the other is received.

Although designed especially for use in grammar grades in accordance with the recommendations of the Committee of Ten and with the practice of many of our foremost schools, this book will be found useful for supplementary work for beginners in demonstrative geometry or for a rapid preliminary drill for such students. Too often the pupil who can recite glibly demonstration after demonstration of geometric principles is unable to make the simplest applications of them, a fact which shows that his work in geometry is not accomplishing its object in developing his mathematical powers. The author trusts that the material of this book will be found useful to teachers in helping them to establish in the minds of learners the habit of making applications of mathematical truths.

Grateful acknowledgments for assistance in revising proof sheets and for valuable suggestions upon the work are due to Prof. O. L. Kelso of the State Normal School, Terre Haute, Ind. and to Prof. S. C. Davisson of the Department of Mathematics of the University of Indiana.

Further suggestions and criticisms from teachers will be gladly received. This little book is sent out with the hope that its use may be as enjoyable to other teachers as the writing of it and its applications in the schoolroom have been to the author.

SUGGESTIONS TO TEACHERS.

THE general method of the pupil's work in this book is that of constructing and inspecting geometric forms and of reporting in the language of mathematics the results of the inspection. His success depends very largely upon the teacher.

Experienced teachers do not need to be reminded that great care is necessary in order that each pupil shall construct carefully, inspect thoroughly, and report exactly.

The following suggestions may be found helpful:

Do not attempt to teach without models the measurements of cubes or other solids. While those measurements are very simple when made upon actual solids, it is beyond the power of the untrained geometric imagination to furnish them. The object of this work being to secure clear thinking, the pupil should not be allowed to form and use false or imperfect mental images of geometric forms.

Rulers, protractors, and dividers, or a substitute for them, are an absolute necessity in this work. The units of metric measurement are to be presented objectively. Very cheap rulers showing decimeter, centimeter, and millimeter can be obtained. A meter-stick marked like a yard-stick with divisions should be made and used. A square meter and its sub-

divisions marked on the floor or wall of the schoolroom will furnish the standard of thinking when the metric units of a problem are squares. A convenient form of protractor will be found at the end of this work, which may be detached without injuring the book. It will serve as a general model in the construction of protractors.

All solutions that use superposition as in G.C.M. and L.C.M. should be actually performed by cutting out and superposing until each child knows perfectly the significance of the operations — but no longer. The skill of the teacher in recognizing the moment when the pupil has gained clear and correct concepts of that which he illustrates, and in stopping the processes of illustration before they degenerate into wearisome and time-eating formalities, is like that of the physician who wisely adjusts his treatment of a case to its varying conditions.

The exercise of the geometric imagination should be encouraged by allowing pupils to substitute mental images for physical solids as soon as the teacher is absolutely sure that those images are accurate and complete. It is evident that the establishment of sympathetic relations between teacher and pupil is a very direct means of gaining this assurance and at the same time of facilitating the progress of the learner.

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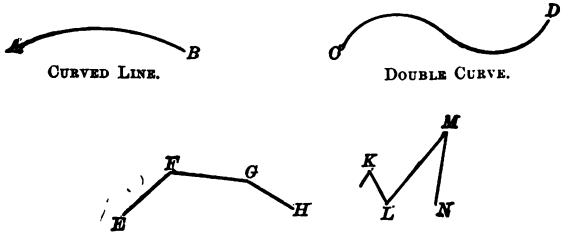
CHAPTER I.

LINES AND ANGLES.



Curved Line. Double Curve.

Broken Lines.

1. What capital letter in print is formed by a straight line and two curves?

2. Which of the capitals is formed by a double curve?

3. Draw a broken line consisting of two straight lines.

4. Name two capital letters each of which is formed by a broken line consisting of two straight lines.

5. Name two capitals each of which is formed by a broken line consisting of three straight lines.

6. Name two capitals each of which is formed by a broken line consisting of four straight lines.

7. What is a broken line?

8. Mark two points *A* and *B*, and connect them by (1) a straight line, (2) a broken line, (3) a curve, and (4) a double curve. Which is the shortest of the connecting lines?

9. Read the following geometric principle and see if it is true :

PRINCIPLE 1. — *A straight line is the shortest distance between two points.*

10. Mark two points C and D on a piece of paper, and find how many straight lines can be drawn from one to the other.

11. How many broken lines can be drawn between C and D?

12. How many curved lines can be drawn between C and D?

13. Can you show the truth of the following statement?

PRINCIPLE 2. — *Between two points only one straight line can be drawn.*

14. If you mark two points on the surface of a ball, and run a straight line joining the points, will the straight line lie on the surface of the ball? Illustrate your answer.

15. If you mark two points on a flat surface, and connect them by a straight line, will the line lie wholly in the surface?

NOTE. — A surface such that if any two of its points are joined by a straight line, the line will lie wholly in the surface, is called a **Plane Surface** or **Plane.**

16. If the waves of the ocean were stilled, would its surface be a plane?

17. Can a ship on the ocean sail in a straight line?

18. Can you draw a straight line on the surface of a ball?

19. Can you draw a curved line on a plane surface? Illustrate.

20. Make an angle by drawing two lines that meet.

NOTE. — The word "line" is used to mean straight line.

21. Find a square corner on the floor. On the wall. On the ceiling. On a page of your book. On your ruler.

22. Two lines which meet so as to make a square corner form a right angle. Make a right angle by drawing along the edges of an object which has a square corner.

23. Draw an acute angle.

NOTE. — An **Acute Angle** is an angle less than a right angle.

24. Draw an obtuse angle.

NOTE. — An **Obtuse Angle** is one that is greater than a right angle.

25. Place the points of two pencils together in such a way as to make a right angle. Keeping the points together, separate the ends of the pencils more. What kind of an angle is formed by the pencils? If the ends were brought nearer than when the pencils formed a right angle, what kind of an angle would be formed?

26. How many angles are formed by the lines of the letter W, and of what kind are they?

27. Give the number and kinds of angles found in the letter N, in T, in X, in Y, in Z.

28. Give the number of each kind of angle found in the letters of the word AWAKE. In your name.

29. Give the number of each kind of angle found in the letters of the phrase THE NEXT TIME.

30. Give the number of each kind of angle found in the sentence YET THEY WILL TAKE THE TAX AWAY.

31. Make two right angles with two lines.

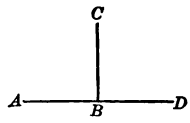
NOTE. — When one line meets another so as to make two adjacent equal angles, the lines are said to be **Perpendicular** to each other and the angles are called **Right Angles.**

32. Make four right angles with two lines.

33. Make one right angle with two lines. Mark its vertex *A*, and the ends of the lines, *B* and *C*.

NOTE. — The point where two lines meet is called the vertex of the angle. In reading an angle, the letter at the vertex should be placed between the other two; thus, the angle $A \begin{matrix} C \\ \diagdown \\ D \end{matrix}$ is read, the angle *CAD* or *DAC*.

34. Name the two adjacent angles at the point *B*. What line belongs to both of those angles?



35. Draw a line *AB* and place an angle *DEF* so that its vertex shall be somewhere on the line *AB*, and no other point of the angle shall touch that line.

36. With three lines make three angles.

37. Draw three lines inclosing a surface. How many angles are thus formed?

NOTE. — A figure bounded by three straight lines is called a **Triangle.**

38. Draw a plane figure bounded by three lines and write its name.

NOTE. — When an inclosed surface is a plane it is called a **Plane Figure.**

39. Can you draw a plane figure on the surface of a ball?

40. Can you draw a plane figure on the surface of a stovepipe?

41. Can you draw a straight line on the surface of a stovepipe?

42. How many lines form the perimeter of a triangle?

NOTE. — The boundary line of an inclosed surface is called its **Perimeter.**

43. How long is the perimeter of a triangle each side of which is 5 inches?

44. Cut three strips of paper, each 7 centimeters long, place them so as to inclose the largest possible triangle, and find the length of its perimeter.

45. Draw a rectangle and show how many lines form its perimeter.

NOTE. — A plane figure each of whose angles is a right angle is called a **Rectangle.**

46. How long is the perimeter of a rectangle of which one side is 10 inches in length and the side adjacent to this, 6 inches?

47. Draw a rectangle whose width is 5 centimeters, and whose length is three times its width, and find the length of its perimeter.

48. Find the perimeter of a rectangle whose width is 6 inches and whose length is 5 inches more than twice its width.

49. Find the perimeter of a rectangle one side of which is 12 inches and an adjacent side of which is 6 inches less than three times as long. The width of this rectangle is what fractional part of its length?

50. Draw a rectangle whose length is 14 inches and whose width is 20 inches less than twice its length. Find the perimeter.

51. How long is the perimeter of a rectangle whose sides are each 8 inches?

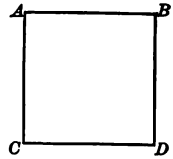
52. What name is given to a rectangle whose sides are all equal?

NOTE. — A plane figure having four equal sides and four right angles is called a **Square.**

53. Is every square a rectangle? Is every rectangle a square? Explain your answers.

54. How long is each side of a square whose perimeter is 28 inches?

55. In the square $ABCD$, what line has the same direction as AB?



NOTE. — Lines which have the same direction are called **Parallel Lines.**

56. What line is parallel to AC?

57. If the perimeter of the square is 24 inches, how far is the line AB from the line CD? Are they everywhere equally distant? If they were both prolonged, would they ever meet?

58. Draw, if you can, two parallel lines which are 3 inches apart in one place and 5 inches apart in another place.

59. Draw two lines perpendicular to each other. What kind of an angle is formed?

60. In the square which illustrates Ex. 55, what lines are perpendicular to CD? To AC?

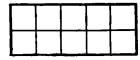
61. How long is the perimeter of a square inch?

62. How many inches are there in the perimeter of a square foot?

63. Place two pieces of paper or wood, each 1 inch square, on a flat surface so that an edge of one shall coincide with the edge of the other, and find the perimeter of the rectangle thus formed.

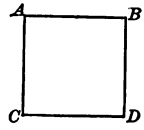
64. Place 5 one-inch squares in a row, their edges coinciding where possible, and find the perimeter of the rectangle thus formed.

65. Place 10 inch squares in two rows as in the cut, and find the perimeter of the rectangle thus formed.



66. Place 25 one-inch squares so as to form a square, and find its perimeter.

67. In the square $ABCD$, which angles are next to the angle A? Which angle is not next or consecutive to A? What kind of a line is the shortest distance from A to D? What geometric principle tells that?



68. Reproduce the square, and draw a diagonal from the point A.

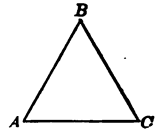
NOTE. — A **Diagonal** of a plane figure is a line drawn from the vertex of one angle to the vertex of another angle which is not consecutive.

69. How many diagonals can be drawn from the point A? What geometric principle applies here?

70. How many diagonals can be drawn in a square? How many can be drawn in a rectangle which is not a square?

71. How many diagonals can be drawn in a triangle? Give the reason for your answer.

72. In the triangle ABC, which is the longer, AC or ABC? What geometric principle states that fact in a general way? Which is the longer, AB or $AC+CB$?



73. Show the truth of the following proposition:

PRINCIPLE 3. — *Any side of a triangle is less than the sum of the other two sides.*

74. Cut three narrow strips of paper, one 10 inches long, the others each 5 inches long, and place them, if you can, so as to make a triangle.

75. Can you make a triangle, one side of which is 12 inches and each of the other sides of which is 6 inches long?

76. Can you make a triangle whose sides are 7 inches, 8 inches, and 9 inches? 4 inches, 5 inches, and 10 inches? Give reasons.

77. If $AB=7$ inches and $BC=5$ inches, how long is AC? What line is the sum of the lines AB and BC?

78. What line is the difference between AC and AB? Between AC and BC?

79. What line equals the sum of AC and CD? Of BD and DC?

80. What line is the difference between AB and AC? Between AD and CD? Between AB and DB?

81. What line is the difference between AB and the sum of AC and CD?

82. $AD+DB-AC=?$

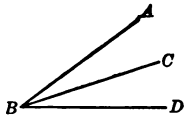
83. $AC+CB-DB=?$

84. $AD+DB-CB=?$

85. $AC+CB-(AD-CD)=?$

86. $AD+DB-(AC+CD)=?$

87. What angle is the sum of the adjacent angles ABC and CBD? What side is common to those angles?



88. If you take the angle CBD from the angle ABD, what angle remains?

89. If one line is 5 times another, and the sum of the lines is 18 inches, how long is each line?

90. Lay off a line 3 inches long on a line 15 inches long and find how many times the longer line contains the shorter one.

NOTE. — A line which is contained in another line a whole number of times is said to measure it.

91. How many times will a line 4 inches long measure a line 32 inches long?

92. How many times will a line 1 inch long measure the length of your desk?

93. How many times will a stick 6 centimeters long measure a stick 48 centimeters long?

94. A coin which measures 3 inches around the edge will revolve how many times in rolling 12 inches?

SUGGESTION. — Roll a coin and find how the distance around its edge compares with the distance it rolls in revolving once.

95. How many times will a wheel whose circumference is 6 feet revolve in running 60 feet?

QUERY. — What is the circumference of a wheel?

96. How long is the tire of a wheel that revolves 9 times in running 45 feet?

97. What is the circumference of a coin that revolves 10 times in rolling 40 inches?

98. What is the length of the longest line that can be laid off a whole number of times on each of two lines, one 6 inches long and the other 8 inches long?

NOTE. — The longest line which exactly measures two or more lines is called their **Greatest Common Measure.**

99. What line is the greatest common measure of two lines, one 15 inches, the other 18 inches?

100. How long is the G. C. M. of a 12-inch line and an 18-inch line?

101. What is the circumference of the largest coin which, in rolling either 9 inches or 12 inches, will make a number of complete revolutions?

102. What is the circumference of the largest wheel which will make a number of complete revolutions in running 20 feet, and also in running 30 feet?

103. What is the circumference of the largest wheel which will make a number of complete revolutions in running 25 feet, and also in running 15 feet?

104. What is the length of the shortest line upon which you can lay off two lines respectively 8 inches and 10 inches a whole number of times? How many times will it contain the longer line?

NOTE. — The shortest line which can be exactly measured by two or more lines is called their **Least Common Multiple.**

105. How long is the L. C. M. of an 8-inch line and a 12-inch line? How many times is it measured by the 8-inch line?

106. Draw the line which is the L. C. M. of a line 10 centimeters and 8 centimeters.

107. Find the shortest line that can be exactly measured by each of two lines, one of which is 9 inches and the other 12 inches.

108. Find the L. C. M. of three lines, respectively 8 inches, 12 inches, and 16 inches.

109. A front wheel of a toy wagon is 20 inches in circumference; a hind wheel is 30 inches in circumference. How far shall the wagon run that both wheels shall have made a number of complete revolutions? Illustrate.

110. A front wheel of a toy wagon is 3 feet in circumference; a hind wheel 4 feet. In traveling what distance will both wheels make a number of complete revolutions, and how many will each make?

111. Describe and illustrate everything which has been explained in the notes of this chapter.

CHAPTER II.

CIRCLES.

NOTE. — A **Circle** is a plane figure whose perimeter is a curved line, every point of which is equally distant from a point within, called the **Center.** The perimeter of a circle is called a **Circumference.** A **Diameter** is a straight line passing through the center and having both ends in the circumference. A straight line from the center to the circumference is called a **Radius.**

1. With a pair of dividers draw a circle with a radius 5 inches long. How long is its diameter?

2. Draw a circle with a radius of 4 inches, and draw several diameters. How long is each diameter?

3. With a radius of $3\frac{1}{2}$ inches draw a circle on a piece of pasteboard. Cut it out and measure the circumference with a tape measure. You will find that it is about 22 inches long. If it were exactly 22 inches, the circumference would be how many times the diameter?

NOTE. — When you study demonstrative geometry, you will find another way of proving the truth of the following theorem:

PRINCIPLE 4. — *The circumference of every circle is 3.1416+, or $3\frac{1}{7}$ (nearly) times its diameter.*

4. What is the circumference of a circle whose diameter is 14 inches?

NOTE. — We use $3\frac{1}{7}$ to express the relation of a circumference to its diameter.

5. What is the circumference of a circle whose radius is 14 inches?

6. What is the circumference of a circle whose diameter is 10 inches?

7. What is the circumference of a circle whose diameter is 28 meters?

QUERY. — How many inches long is a meter?

8. The radius of a circle is 2 meters. Find its circumference in centimeters.

9. The diameter of a circle is 3 meters. Find its circumference in centimeters.

10. The circumference of a circle is 44 inches. Find its diameter.

11. The circumference of a circle is 33 inches. What is its radius?

12. The circumference of a circle is 22 inches. Find its diameter.

13. The diameter of a nickel of the issue of 1866 is 2 centimeters. What is the circumference?

14. A horse is tied to a stake by a rope 7 feet long. Find the length of the longest path he can travel in going once around the stake.

15. If the outer edge of a merry-go-round is $10\frac{1}{2}$ feet from the center, how far does John travel when he sits at the outer edge and makes one revolution?

16. If he faces forwards and keeps his hands at his sides, which hand travels the faster, his right or left hand?

17. What is the length of the longest stick that can lie wholly on a round table which is 11 feet in circumference?

18. What is the longest line that can be drawn across a round flower bed whose circumference is 12 feet 10 inches?

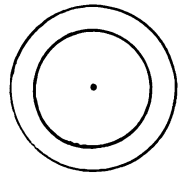
19. Mary has embroidered a round mat 1 foot 10 inches in circumference. If it were cut into two equal parts, what would be the length of the straight edge of each?

20. Using "semi" to denote "half," how would you define a semicircle? A semicircumference?

21. Draw a semicircle, and write the names of its bounding lines.

22. The radius of a certain circle is 2 feet 11 inches. By how much does the curved boundary line of one of its semicircles exceed the straight boundary line?

23. The smaller circle is placed upon the larger, so that their centers coincide. If the diameter of the larger circle were 10 inches, and that of the smaller 7 inches, what would be the width of the circular ring included between the circumferences?



24. If you cut a circle whose diameter is 7 inches from the center of a circle whose diameter is 11 inches, what is the width of the circular ring that remains?

25. Find the length of the arc which is $\frac{1}{6}$ of the circumference of a certain circle whose diameter is 42 inches.

NOTE. — An **Arc** is a part of a circumference. Compare arc and arch. Is a semicircumference an arc?

26. One arc of a circle is 5 times the remaining arc. The circumference is 120 meters. How many meters long is each arc?

27. Find the length of an arc which is $\frac{2}{5}$ of the circumference of a circle whose radius is 14 inches.

28. An arc is 9 times the remaining arc. The diameter of the circle is 7 feet. Find the length of each arc.

QUERY. — How long is the circumference?

29. If a man is on a plain where he can see 3 miles in every direction, what is the length of the line which bounds the part of the plain over which he can see?

30. Draw an arc and connect its ends by a straight line. The straight line is called a chord.

NOTE. — The word "subtend" is derived from two Latin words, meaning "to stretch under." Would you say that the arc subtends the chord, or the chord subtends the arc?

31. Draw a segment, and write upon it the names of its bounding lines.

NOTE. — The part of a circle included between a chord and its arc is called a **Segment.**

32. Is a semicircle a segment? Give reasons.

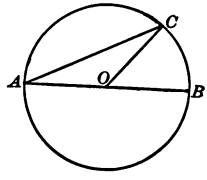
33. How many segments are formed in a circle by drawing a chord? How many arcs?

NOTE. — A segment greater than a semicircle is called a greater segment. A segment less than a semicircle is called a less segment.

34. Can three greater segments be cut from a circle?

35. Which is the longer line, an arc or its chord? State the geometric principle which applies.

36. In the circle whose center is O, which is the longer, the straight line AB or the sum of the lines AO and OC? The straight line AC or the sum of the lines AO and OC? The line AC or AB? Give reasons.



37. Can you draw a chord which is not a diameter of the circle and yet is equal to a diameter?

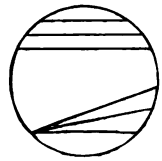
38. Can you see the truth of the following statement?

PRINCIPLE 5. — *A diameter is longer than any other chord.*

39. What is the longest line that can be drawn on the head of a drum 40 inches in circumference?

40. An object which has a circle for its base, and tapers to a point, is called a **Cone**. Draw or make a cone. What is the length of the longest line that can be drawn across its base if the circumference is 121 millimeters?

41. Draw in a circle several parallel chords on the same side of the center of the circle. Cut out and fold the circle so as to place one chord upon the other. Which is the longest?



42. Draw in a circle several chords from the same point. How do they compare in length?

43. Can you show the truth of the following statement?

PRINCIPLE 6. — *In the same circle or in equal circles the greater of two chords is at a less distance from the center.*

44. With a given point as a center and with any radius describe a circle and cut it out. Draw a diameter, fold the circle along that line, and see if the two surfaces coincide. Does the diameter bisect the circle?

NOTE. — To bisect means to divide into two equal parts.

45. Draw two parallel chords in a circle, fold the circle so that the two halves of one chord shall coincide. Will

the halves of the other chord coincide? Will the two arcs intercepted by the ends of the chords also coincide?

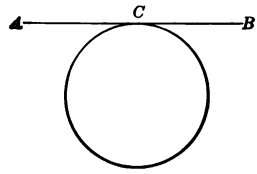
46. Show by another circle the truth of the following statement:

PRINCIPLE 7. — *Parallel chords intercept equal arcs.*

47. Draw a circle and a straight line touching the circumference at one point.

NOTE. — A straight line, which, however prolonged, touches a circumference at only one point is called a **Tangent,** and the point at which it touches is called the point of tangency.

48. Draw two parallel tangents and join their points of tangency. Try to draw three parallel tangents to the same circle.



49. Draw three tangents to a circle, placing two of them upon one semicircumference and the third on the other. Prolong them until they inclose a surface. What figure is formed?

NOTE. — A plane figure whose sides are all tangent to a circle is said to be circumscribed about the circle and the circle is said to be inscribed in the figure.

50. Try to draw three tangents to a circle in such a way that when prolonged they form a triangle which is not circumscribed about the circle.

51. Draw four tangents to a circle, placing two of them on each semicircumference, and prolong them until they inclose a surface. What figure is formed?

NOTE. — A plane figure bounded by four straight lines is called a **Quadrilateral.**

52. Draw two parallel tangents to a circle and another pair of parallel tangents perpendicular to the first, and

write the name of the figure thus circumscribed about the circle.

53. Can you circumscribe about a circle a rectangle which is not square ?

54. Describe everything which is explained in the notes of this chapter, forming a clear mental picture of that which you describe.

CHAPTER III.

ARCS AND ANGLES.

NOTE TO TEACHER. — A fan opened so as to form different angles is very useful in illustrating the subject of this chapter, parts of the edge of the fan being used to represent arcs.

The circumference of every circle may be considered as divided into 360 equal parts called degrees, marked °.

1. Draw a semicircumference and tell how many degrees there are in it.

2. How many degrees in the circumference formed by the edge of a teacup? How many in the Equator? Which is the longer, a degree of the circumference of the teacup or a degree of the Equator?

3. An arc of one degree is what fractional part of the circumference?

4. How long is an arc of one degree of a circumference which is 1800 inches?

5. How many degrees are there in the arc described by the end of the minute hand of a clock in half an hour?

6. How many degrees are there in a quadrant?

NOTE. — A **Quadrant** is one fourth of a circumference.

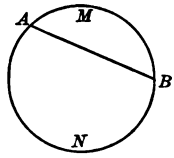
7. In 3 hours, how many degrees are described by the hour hand of a clock? How many by the minute hand in 10 minutes?

8. How many degrees are described by the second hand of a watch in $\frac{3}{4}$ of a minute?

9. How many degrees are described by the minute hand in 20 minutes?

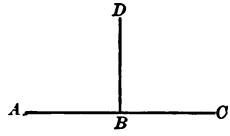
10. John and James started from the same point in a circular race track, and ran around the track in opposite directions until they met. John ran $\frac{5}{12}$ of the distance. How many degrees were there in the arc which each passed over?

11. The arc ANB is twice the arc AMB. How many degrees are there in each?



12. If a circumference is divided by a chord so that the greater arc is 3 times the less, how many degrees are there in each arc?

13. If the angles ABD and DBC are equal, the lines BD and AC are in what relative position? What kind of angles are ABD and DBC?



SUGGESTION. — See Chap. 1, Ex. 31, Note.

14. At what times of day, when the minute hand is at XII., do the hands of a clock form a right angle?

15. How many degrees are there in the angle formed by the hands of a clock at 3 o'clock?

NOTE. — A right angle is an angle of 90 degrees.

16. How many degrees are there in the angle formed by the hands of a watch at 1 o'clock?

17. How many degrees are there in the angle formed by the hands of a clock at 4 o'clock in the morning? At 4 in the afternoon?

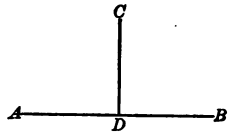
18. How many degrees are there in the angle formed by the hands of a clock at 10 o'clock?

19. How many degrees are there in the angle formed by the hands of a watch at 8 o'clock?

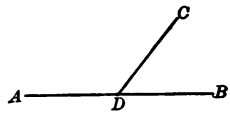
20. At what time of the day, when the minute hand is at 12, do the hands form a straight angle?

NOTE. — An angle equal to two right angles is called a **Straight Angle.** Each of the lines which form it is the prolongation of the other.

21. Draw the line CD perpendicular to AB, as in the cut. How many degrees are there in the two angles ADC and CDB?



22. If the line CD were inclined as in the cut, how many degrees would there be in the sum of ADC and CDB? If ADC is 120°, how many degrees is CDB?



QUERY. — As one angle grows larger and the other smaller, does the number of degrees in their sum change?

PRINCIPLE 8. — *The sum of the two adjacent angles formed by one straight line meeting another is equal to two right angles.*

23. What is the supplement of an angle of 100°?

NOTE. — Two angles which together equal 180° are said to be **Supplements** of each other. Two arcs which together equal a semicircumference are **Supplements** of each other.

24. What is the supplement of a right angle?

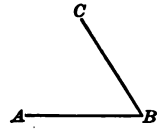
25. What is the supplement of an angle which is $\frac{4}{5}$ of a right angle?

26. What is the supplement of an arc of 75° ?

27. What is the supplement of an angle of 47° ?

28. What is the supplement of an arc of 68° ?

29. If the angle ABC is 50°, how many degrees are in the angle formed by prolonging the line AB from the point B? By prolonging CB from the point B?



30. Draw by means of your protractor an angle of 70°, and show how many degrees its supplement has.

31. Measure the angles formed by the branching from the main stem of different plants, and determine the supplements of those angles.

32. How many degrees has an angle whose supplement is 3 times as large as the given angle ?

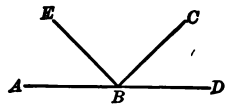
33. How many degrees has an angle whose supplement is 95° ? 75° ? 1° ? $5\frac{1}{2}$° ?

34. How many degrees are there in an angle which is 4 times its supplement ?

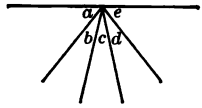
35. Can a right angle have an obtuse or an acute angle for its supplement ? Give reasons.

36. Is the supplement of an obtuse angle obtuse or acute ?

37. How many degrees are there in the sum of the angles ABC and CBD? Is the number of degrees in the angle ABC changed by drawing the line EB? How many degrees are there in the sum of the angles ABE, EBC, and CBD?



38. $a + b + c + d + e$ = how many degrees? Give reasons.



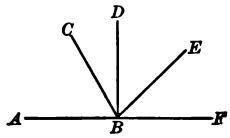
PRINCIPLE 9. — *The sum of all the angles formed at a given point on the same side of a straight line is equal to two right angles or 180°.*

39. Angle DBF is a right angle.

Angle DBE = EBF.

Angle ABC = 2 times CBD.

How many degrees are there in each angle?



40. Place the vertex of a right angle at a point in the line AB so that neither of its sides shall coincide with that line. How many degrees are in the sum of the two angles formed by AB and the sides of the right angle?

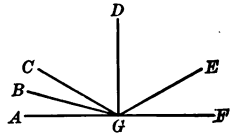
41. Angle AGD = a right angle.

Angle DGE = 2 times angle EGF.

Angle CGD = angle DGE.

Angle AGB = angle BGC.

How many degrees are there in each angle?

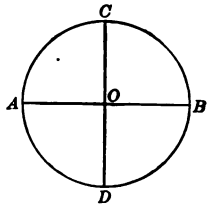


42. How many degrees has each of three angles formed at a given point on the same side of a straight line if the first contains 5 times as many degrees as the second, and the third contains 4 times as many as the second? Illustrate.

43. How many degrees are there in all the angles formed on both sides of the line AB at the point C? At any other point in the line AB?

44. How many degrees are there in all the angles formed around a common point?

45. AB and CD are diameters crossing at right angles. How many degrees are there in each of the four arcs intercepted by them?

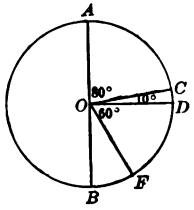


SUGGESTION. — Cut out a similar figure and, folding it, place the parts one upon another and see if they coincide.

46. If the angle AOC is bisected by a radius, how many degrees has each of the angles thus formed? How many has each of the arcs thus formed? Illustrate, using a protractor, or folding.

NOTE. — It will be seen that the number of degrees in the arc is the same as the number of degrees in the angle at the center, whose sides touch the ends of the arc, or, as is said, "the angle is measured by the intercepted arc."

47. How many degrees are there in arc AC? In arc CD? In arc DF? In arc FB?



PRINCIPLE 10. — *An angle at the center of a circle is measured by the arc intercepted by the radii which form it.*

NOTE. — An angle at the center is called a **Central Angle.**

48. Draw a circle and make an angle of 120° by radii. The arc intercepted by the radii is what part of the circumference?

49. An arc intercepted by two radii is $\frac{1}{5}$ of the remaining arc of the circumference. What angle is formed by the radii?

50. A certain circumference consists of two arcs, one of which is 8 times the other. How many degrees are there in the angle measured by the smaller arc? Illustrate.

51. A quadrant is divided into two arcs, one of which is 3 times the other. How many degrees are there in each angle measured by them?

NOTE. — Two arcs whose sum is a quadrant are said to be **Complements** of each other. Two angles whose sum is a right angle are said to be **Complements** of each other.

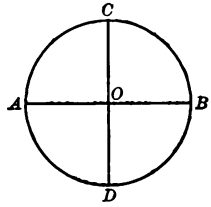
52. What is the complement of an angle of 75°? Of $8\frac{1}{2}°$?

53. Draw the complement of an angle of 40°, and the complement of an arc of 50°.

54. Which is the greater, an angle or its complement? Explain. Which is the greater, an angle or its supplement?

55. By how much does the supplement of an angle of 70° exceed its complement? An angle of 80°? Any angle of less than 90°?

56. If *AB* and *CD* are perpendicular to each other, what part of the circle is the plane figure *OCB*? Cut out and fold a circle and show the truth of your answer. What are the boundaries of *OCB*? The arc *CB* is what part of the circumference?



NOTE. — A surface inclosed by two radii and their intercepted arc is called a **Sector.**

57. Draw a circle, divide it into six equal sectors by making angles of 60° at the center, and show what part the arc of each sector is of the circumference.

58. The arc of a sector which is $\frac{1}{8}$ of a circle is what part of the circumference? How many degrees has its angle?

59. What have you ever eaten whose upper surface could be represented by a sector?

60. Make a sector whose angle is 40° and whose straight edges are 7 inches. What part of a circle is it? Find the length of the circumference of the circle. The length of the arc. The length of the perimeter of the sector.

61. A semicircle is divided into two sectors, the arc of one being 4 times that of the other. How many degrees are there in the arc of each sector? The smaller sector is what part of the semicircle?

62. A sector whose arc is 150° is divided into two sectors, the arc of one being 3 times that of the other. How many degrees are there in the angle of each? Each sector is what fractional part of the circle?

63. How long is the perimeter of a sector of 120° in a circle whose radius is $10\frac{1}{2}$ feet?

64. Find the difference between the length of the curved boundary line and the sum of the two straight boundary lines of a sector of 150° in a circle whose diameter is $17\frac{1}{2}$ inches.

65. If a round pie is cut into six equal pieces, how many degrees has the arc formed by the curved edge of the crust of each piece? How many degrees has the angle formed by the straight sides of each piece?

66. How many times will an arc of 20° be contained in an arc of 80° of the same circumference?

67. How many times will an arc whose length is 20 inches be contained in a circumference whose length is 5 feet.

68. Make a sector of 90° and show by folding how many times it will contain a sector of 45° of the same circle.

69. A wheel which has 6 cogs is geared with one that

has 60 cogs. How many times will the small wheel turn while the large one turns once?

SUGGESTION. — Examine clockwork.

70. What name is given to a quantity which measures another quantity a whole number of times? To the quantity which is an exact measure of two or more quantities? To the largest quantity which exactly measures two or more quantities?

71. How many degrees are there in the arc which is the G. C. M. of an arc of 70° and one of 90° degrees in the same circle or equal circles?

72. One arc of a circle is 21 inches; another arc of the same circle is 28 inches. Find the length of the arc which is their greatest common measure.

73. What sector is the G. C. M. of two sectors of the same circle, one of 60°, the other of 150°?

74. Draw an angle of 40° and another of 50° with lines 5 inches long, and show how many degrees there are in their G. C. M.

75. What name is given to a quantity which contains another quantity a whole number of times? To the quantity which is an exact multiple of two or more quantities? To the least quantity which is an exact multiple of two or more quantities?

76. What arc is the L. C. M. of an arc of 10°, and of an arc of 15° of the same circle?

77. How many degrees has the arc of the sector which is the L. C. M. of two sectors of the same circle, the arc of one being 8° and of the other 6°?

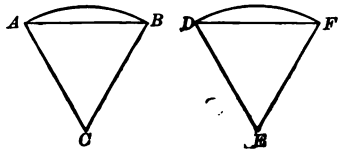
78. What is the L. C. M. of an angle of 15°, and of an angle of 25°?

79. Find the L. C. M. of an arc of 90° and one of 120° of the same circle.

80. A front wheel of a wagon is 21 inches in diameter, a hind wheel 28 inches in diameter. How far can the wagon run that both wheels may make a number of complete revolutions, and how many will each make?

81. Show that a semicircle is a sector and a segment.

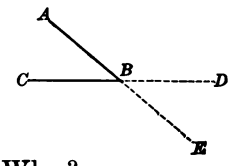
82. If the sector ABC is superposed upon the equal sector DEF so that the point A coincides with the point D, B with F, and C with E, would the chord AB coincide with the chord DF? Can you draw a straight line between the points D and F that would not coincide throughout its whole extent with the line DF? Quote a geometric principle which is applicable.



83. Divide a circumference into four equal arcs by diameters. Join the extremities of each arc by a chord. Superpose and show the truth of the following principle:

PRINCIPLE 11. — *In the same circle or in equal circles equal arcs are subtended by equal chords.*

84. Given the angle ABC 40°; how many degrees has the angle formed by prolonging CB to D? By prolonging AB to E? Which angle is the greater, ABD or CBE? How many degrees are there in the angle DBE? Why?



85. Looking along the line *ABE*, what angle is the supplement of *ABD*? Looking along *CBD*, what is the supplement of *ABD*? Are those angles then equal?

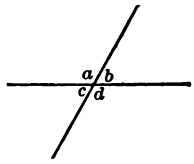
86. If $ABD = 150°$, what is *DBE*? What is *ABC*?

87. If the angle *CBE* equaled 130°, how many degrees would there be in each of the other angles?

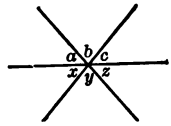
88. Draw two intersecting lines and show which angles are equal.

PRINCIPLE 12. — *If two straight lines intersect, the vertical or opposite angles are equal.*

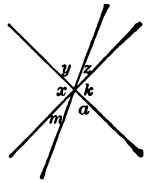
89. Angle a = twice angle b. Find angles a, b, c, d.



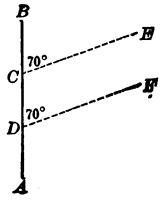
90. Angle $a = 50°$. Angle $c = 50°$. Find each of the others.



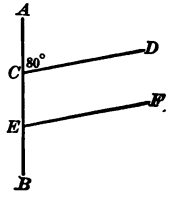
91. Angle $x = 90°$. Find each angle around the common vertex. Angle a = 3 times angle m.



92. Draw a line *AB*, and at some point, as *C*, make an angle of 70°. At some other point, as *D*, make another angle of 70°. Since *CE* and *DF* diverge from *AB* at the same angle, will they lie in the same direction? Are they parallel? How many degrees are there in the angle *ECD*? In *FDA*?



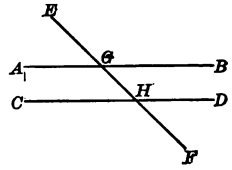
93. Draw two parallels branching from another line AB, making the angle ACD 80°. How many degrees has the angle CEF?



SUGGESTIONS. — 1. Cut out angle ACD and superpose it upon angle CEF.

2. Measure the angles with a protractor.

94. How many angles are formed by a transversal to two parallels?



NOTE. — A line crossing or meeting two parallels is called their **Transversal.** The angles outside the parallels are called **Exterior Angles**; those within the parallels are called **Interior Angles.**

95. Name the interior angles on the right side of the transversal. On the left side.

96. Name the exterior angles on the right side of the transversal. On the left side.

NOTE. — Angles on different sides of the transversal which are not adjacent are called **Alternate Angles.**

97. Name the two pairs of alternate interior angles.

98. Name the two pairs of alternate exterior angles.

NOTE. — Two angles which are on the same side of the transversal, one interior and the other exterior, but not adjacent, are called **Opposite Interior and Exterior Angles.**

99. Angle EGB being an exterior, what is its opposite interior angle? Name the exterior angle opposite to angle AGH. Name the exterior angle opposite to angle HGB. Name the interior angle opposite to angle AGE.

100. Can you draw two parallels cut by a transversal

in such a way that an exterior angle shall be 80° and its opposite interior angle shall be 70°?

101. Reproduce the figure in Ex. 94, changing it so as to make angle EGB 60°. How many degrees are there in angle GHD?

PRINCIPLE 13. — *When two parallel lines are cut by a transversal, the opposite interior and exterior angles are equal.*

102. If angle EGB is 50°, how many degrees are there in the angle BGH? Quote the geometric principle by which you can tell the number of degrees in angle BGH when the number in angle EGB is given.

103. Angle EGB being 50°, how many degrees has the angle AGH? Quote the geometric principle.

104. Angle EGB being 50°, how many degrees has angle AGE? Angle GHD? Quote the geometric principles.

105. If angle AGE were 100°, how many degrees would there be in each of the eight angles formed by the two parallels and their transversal?

106. If angle AGE is 3 times the angle EGB, how many degrees are there in each angle formed by the three lines?

107. Draw a horizontal line and mark two points in it. At these points draw lines crossing the horizontal in the same direction, making an angle of 75° with it. How many degrees has each angle formed by the three lines? How does the figure differ from that given in Ex. 94?

108. Draw two parallels and a transversal so that an exterior angle shall be 4 times its adjacent interior angle. How many degrees has each of the eight angles?

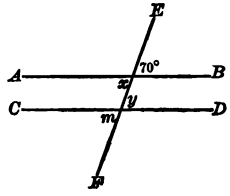
109. John and William with their sister Mary went on an excursion. Their father provided each of his children with money so that John's money equaled Mary's money and William's money equaled Mary's money. Which of the children had the more money, John or William?

110. If the following axiom seems true to you, illustrate it by three angles.

AXIOM 1.

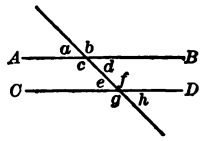
Things which are equal to the same thing are equal to each other.

111. AB and CD being parallel, how many degrees are there in angle y? Give the reason. How many in angle x? Give the reason. Which is the greater of the two alternate interior angles, x or y? Which is the greater of the two alternate exterior angles, the angle which is 70° or angle m?

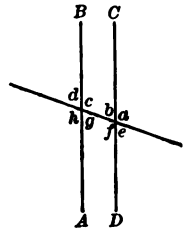


PRINCIPLE 14. — *When two parallels are cut by a transversal, the alternate interior angles are equal.*

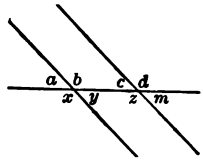
112. AB and CD are parallel. The angle f being twice the angle e, how many degrees has the angle c? If angle a were a right angle, how many degrees would there be in each of the angles formed by the three lines?



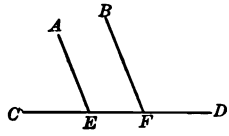
113. If the transversal is perpendicular to one of two parallels, what angles will it form with the other? Illustrate.



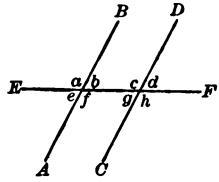
114. If $b = 70°$, how many degrees are there in f? How many in g? If $a = 110°$, how many degrees are there in h? How do b and g compare?



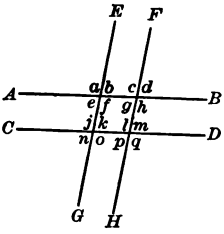
115. If angle d is 135°, how many degrees are there in each of the remaining seven angles?



116. Let the parallels AE and BF meet the line CD. If the angle $BFD = 112°$, how many degrees has each of the remaining angles formed by the three lines?

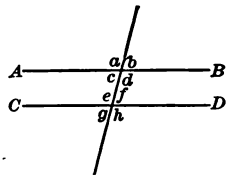


117. AB and CD are parallel. Angle $b = 63°$. How many degrees are there in each of the other seven angles formed by the three lines?



118. Let the parallels AB and CD be crossed by the parallels EG and FH. Angle $a = 100°$. How many degrees has each of the other 15 angles? How many are there in the sum of angle f and angle k? How many degrees are there in the sum of angle g and angle l? In the sum of angle h and angle m? In the sum of angle o and angle p? In the sum of angle e and angle j? In the sum of angle b and angle c?

119. AB and CD are horizontal lines.



If $b = 75°$, how many degrees are there in the sum of d and f?

If $b = 86°$, how many degrees are there in the sum of d and f?

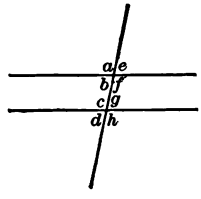
If $b = 85°$, how many degrees are there in the sum of d and f?

If $b = 77\frac{1}{2}°$, how many degrees are there in the sum of d and f?

120. When two parallel lines are crossed by a transversal, what is the sum of the two interior angles on the same side of the transversal?

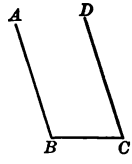
PRINCIPLE 15. — *When two parallels are cut by a transversal, the sum of the interior angles on the same side is equal to two right angles.*

121. When the angle f is twice the angle g, how many degrees has each of the interior angles?

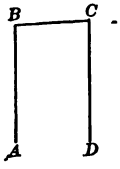


122. When angle f is 150°, how many degrees has each of the angles formed by the three lines?

123. AB and CD are parallel. Angle $ABC = 107°$. Find angle BCD.



124. AB and CD are parallel. Angle $BCD = 86°$. Find angle ABC.



CHAPTER IV.

CUMULATIVE REVIEW NO. 1.

1. Define each term in the following classification:

$$\text{Angles} \begin{cases} \text{Right.} \\ \text{Oblique} \begin{cases} \text{Acute.} \\ \text{Obtuse.} \end{cases} \end{cases}$$

2. Draw two adjacent angles, two adjacent supplementary angles, two supplementary angles that are not adjacent, two adjacent complementary angles, two complementary angles that are not adjacent.

3. Can you draw two unequal vertical angles?

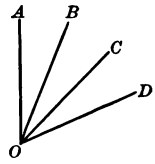
4. Define perimeter. What special name is given to the perimeter of a circle?

5. What is the circumference of a circle whose radius is 35 centimeters?

6. How many millimeters are there in the sum of the circumferences of 7 nickels?

7. The circumference of a circle whose diameter is 35 millimeters is what fractional part of the circumference of a circle whose diameter is 63 millimeters?

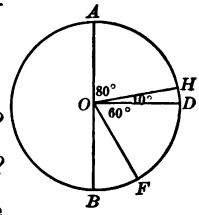
8. The circumference of a circle whose diameter is 14 centimeters is how many times the circumference of a circle whose diameter is 7 centimeters?



9. Name the angle which is the sum of the angles AOB and BOC; the difference of AOD and AOC; the difference of AOC and AOB.

10. Angle BOC + angle COD = ?

11. Angle AOC + angle COD − angle AOB = ?



12. What sector is the sum of the sectors FOD and DOH?

13. $AOD + DOB - BOF$ = ?

14. $BOH + HOA - (BOF + FOH)$ = ?

15. $AOF + FOB - (AOD - AOH)$ = ?

16. How many degrees are there in the complement of the angle formed by the hands of a clock at 2 o'clock?

17. How many degrees are there in the supplement of the angle formed by the hands of a clock at 4 o'clock?

18. Make an angle of 50° whose sides are each 3 in. long. Prolong the sides until they are each 5 in. long. Have you increased the angle?

NOTE. — An angle is the amount of divergence of two lines that meet or that will meet if sufficiently prolonged.

19. Examine a compass. How many degrees are included in the arc between the points north and southeast? How many between E. and S.E.? Between N.E. and S.W.? Between S.W. and N.W.?

20. Compare the angles formed by the branches and twigs of different kinds of trees.

21. Apply your protractor to the lines of plaid goods, and find the number of degrees in the angles formed by them.

22. Measure angles in patterns of wall paper, carpets, etc., until you can estimate the number of degrees in an angle quite accurately.

23. If we represent the length of a line by x, how shall we represent the length of a line twice as long? How shall we represent the sum of the lines? If x were 7 inches, what would $3x$, their sum, equal? If x were 10 inches, what would their sum equal? How long is the line x if $3x = 24$ inches?

NOTE. — A statement of the equality of quantities is called an **Equation**, as $3x + 5 = 23$.

24. The sum of two lines is 15 inches and one line is twice the other. Find the length of each.

SOLUTION. — If we let x equal the shorter line, $2x$ would equal the longer, and their sum would be $3x$. We have the equations

$x + 2x = 15$, then $x = 5$, the shorter line,

then $3x = 15$, then $2x = 10$, the longer line.

25. Make an equation about the length of two lines, one of which is 7 times the other, and their sum, 40 inches. Represent the shorter line by x. What represents the longer line? Their sum? Find the length of each line.

NOTE. — Equations are used in solving problems referring to any kind of units. Learn to use them by putting the statement of your problems into equations whenever possible, as they will make your work much easier as soon as you learn their use.

26. Make an equation, and solve it, about the number of marbles of John and James. John has 5 times as many as James and the sum of their marbles is 18.

27. The sum of two angles is 150° and the greater is 4 times the less. How many degrees are there in each?

QUERY. — What shall x equal?

28. The sum of three lines, AB, BC, and CD, is 84 inches. BC and CD are each 3 times AB. Find the length of each.

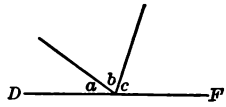
SUGGESTION. — Let $x = AB$
then $3x = BC$
and $3x = CD$
$7x = 84$

from which we find the required values.

29. The sum of three lines, AB, CD, and EF, is 81 inches. CD is twice EF and AB is 3 times CD. Find the length of each.

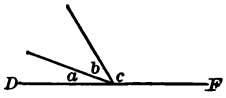
QUERY. — If x equals EF, what will CD equal? AB?

30. Angle b = twice angle a.
Angle $c = b$. How many degrees has each angle, DF being a straight line?

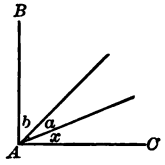


NOTE. — In an equation those quantities whose values are known are called "known quantities." In Ex. 30 the known quantity is 180°.

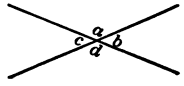
31. Angle b = twice angle a, angle $c = 3$ times angle b. How many degrees has each angle?



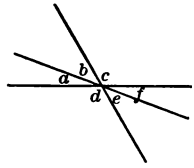
32. BA is perpendicular to AC.
Angle a = angle x.
Angle b = twice angle a. How many degrees has each angle?



33. Angle $a = 3$ times angle b. Find the number of degrees in each of the four angles.



34. Angle b = twice angle a, angle c = twice the sum of angle a and angle b. How many degrees has each of the six angles?

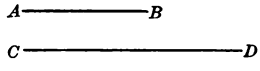


QUERY. — If we know the number of degrees in angle a, how can we tell the number in angle f?

35. If we represent a line by x, how may we represent a line 3 inches longer? If their sum is 15 inches, what equation would state that fact? Given the equation $2x+3=15$. Find the value of x.

If $2x$ and 3 equals 15, is it not true that $2x$ without the 3 equals 15 lacking 3? If $2x=12$, $x=6$, the shorter line, and $x+3=9$, the longer line.

36. The sum of the lines AB and CD is 17 inches. CD is 5 inches longer than AB. Find the length of each.



SUGGESTION. — Given the equation $2x + 5 = 17$. If we transpose the 5 on the left side of the sign of equality to the other side, changing the sign, we shall get the same result as by the reasoning of the previous example, $2x = 17 - 5$.

37. Find the value of x when $3x + 7 = 37$.

Find the value of x when $4x - 5 = 35$.

QUERY. — If $4x$ lacking 5 equals 35, what is the value of $4x$ not lacking anything?

NOTE. — A quantity may be transposed from either side of the equation to the other without destroying the truth of the equation, if the sign of the quantity is changed.

38. How long is the line x if $4x-7=3x+5$, the known quantities referring to inches?

SUGGESTION. — Transpose the x's to the left side, the known quantities to the right. When you change the side, change the sign.

39. Find the length of the line y if $2y+2=y+120$, the known quantities representing inches.

40. Find the length of the line n if $5n+7=4n+16$.

41. Find the length of the line x if $3x-7=x+13$.

42. Find the length of the line x if $7x-8=3x+16$.

43. If a line 8 inches long is represented by x, how shall we complete the following and make a true equation? $7x-10=5x+?$

44. Complete $8x-3x=?$ x being 11 centimeters.

45. Complete $11x-21=?$ x being 3 decimeters.

46. Complete $3x+9+8x-3=4x+?$ x being a line 4 inches.

47. The sum of two lines is 61 inches and the greater is 5 times the less and 7 inches more. Find the length of each.

48. The shorter of the two lines in Ex. 47 is 9 inches. Substitute 9 for x in the equation $6x+7=61$, and see if the equation is true.

NOTE. — The process of substituting a value for x and showing the equation to be true is called **Verification**. It is a convenient way of proving the work.

49. The sum of two angles is 100° and the greater is 40° more than the less. Find each and then verify your work.

50. Divide a right angle into two angles, one of which is 40° more than the other.

51. How many degrees are there in the angle whose complement contains 40° more than the angle itself? Verify.

52. How many degrees are there in the angle whose complement contains 30° more than the angle itself?

53. Divide an arc of 120° into two arcs, one of which contains 20° more than the other.

54. A circular flower bed is 48 feet in circumference. It is bordered a part of the way with pinks and the rest of the way with mignonette. The edge planted with pinks is 3 times as great as that planted with mignonette. Find the number of feet of the edge given to each.

55. John and James started from the same place and ran round a circular race track 120 meters in circumference until they met. John ran 20 meters more than James. How far did each run?

QUERY. — Can you think how the track and the boys running on it would look? It is necessary to form clear pictures in your mind of things described in your problems.

56. John, James, and William ate a round pie 22 inches in circumference. The curved edge of John's piece was 2 inches longer than that of James's and that of James's was 3 inches longer than that of William's. Give the length of curved edge of each piece. What fractional part of the whole pie did each boy eat?

57. The same boys ate another round pie 28 inches in circumference. William's piece was twice as large as James's, and James's was twice as large as John's. Give the length of the curved edge of each piece.

58. Albert, George, and Charles built a fence around a circular lot 300 feet in circumference. Albert built twice as much as George and Charles built 3 times as much as George. How many feet did each build?

59. Mary, Jennie, and Anna embroider the edge of a round tidy 42 inches in circumference. Mary works twice as much as Jennie, and Anna works 3 times as much as Jennie. How many inches of the edge does each embroider?

60. Tom, Fred, and Will whitewashed the fence around a circular lot 60 feet in circumference. Fred whitewashed 3 times as much as Tom, and Will 4 times as much as Tom. How many feet of fence did each whitewash?

61. Mary, Jennie, and Anna decorate a round table 44 inches in circumference. Mary decorates 8 inches more of the edge than Jennie, and Anna twice as many as Mary. How many inches does each decorate?

SUGGESTION. — $2(x + 8) = 2x + 16$.

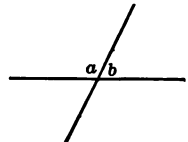
62. Divide a quadrant into two arcs, one of which is 10° more than the other.

63. Three angles are formed at the same point on the same side of a straight line. The first is 20° greater than the second, and the second is 30° greater than the third. How many degrees has each? Represent the angles.

64. Three angles are formed around a common point. The first contains 15° more than the second, and the second 30° more than the third. How many degrees has each? Represent.

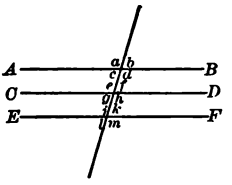
65. A long side of a rectangle whose perimeter is 80 centimeters, is 5 centimeters more than 4 times a short side. Find each side.

66. If the angle a is 20° less than twice the angle b, how many degrees are there in each angle?

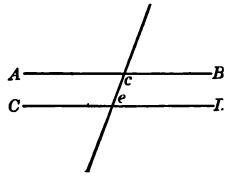


67. What are parallel lines?

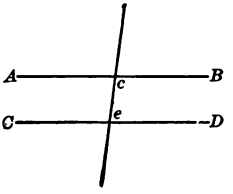
68. Angle *a* being 107°, how many degrees are there in each of the other angles formed by the transversal and three parallels?



69. How many degrees are there in each angle if angle *c* is 50° more than angle *e*?



70. How many degrees are there in each angle if angle *c* is 30° less than 3 times angle *e*?



CHAPTER V.

RECTANGLES.

NOTE. — A **Polygon** is a plane figure bounded by straight lines. When a polygon is bounded by four straight lines it is called a **Quadrilateral.** When the opposite sides of a quadrilateral are parallel it is called a **Parallelogram.** When a parallelogram has four right angles it is called a **Rectangle.** When the sides of a rectangle are all equal it is called a **Square.** A parallelogram which is not a rectangle is called a **Rhomboid.**

1. Draw a polygon. Is a sector a polygon?

2. Draw a quadrilateral.

3. Draw a parallelogram.

4. Draw a rectangle.

5. Draw a square and show that it is entitled to six different names.

6. Draw a rhomboid and show to how many names it is entitled.

7. To how many names is a rectangle entitled?

8. To how many names is a triangle entitled?

9. Construct a rectangle having 3 rows of 1-inch squares, each row containing 8 squares. How many square inches does it cover?

NOTE. — The amount of surface which a figure covers is called its **Area.**

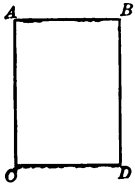
10. What is the area of a rectangle consisting of 2 rows of 1-inch squares, each row containing 4 squares?

11. What is the area of a rectangle consisting of 3 rows of 1-inch squares, 7 squares in a row? How long is its perimeter?

12. Draw a rectangle whose base is 5 inches and altitude 4 inches. How many square inches in area is it?

NOTE. — The lower base of a rectangle is either one of the sides upon which it may be supposed to stand, the upper base is the side opposite the lower base, and the altitude is the distance between the bases, measured perpendicularly.

13. In the rectangle *ABCD*, what line is the lower base? The upper base? What lines represent the altitude? If we turn the rectangle and consider *AC* the lower base, what lines represent the altitude?



14. Show the truth of the principle:

PRINCIPLE 16. — ***The area of a rectangle is equal to the product of its base and altitude.***

15. What is the area of a rectangle whose base is 12 inches and altitude half as much?

16. Find the area of a rectangle whose base is 5 centimeters and altitude 8 centimeters.

17. Base $= 7\frac{1}{2}$ inches; altitude $= 5$ inches; required the area of rectangle.

18. Base $= 16\frac{2}{3}$ inches; altitude $= \frac{1}{2}$ of base; required the area of rectangle.

19. Base = twice the altitude; sum of base and altitude = 15 inches; find the area of rectangle.

SUGGESTION. — Let x = altitude.

20. Base = 3 times the altitude; sum of base and altitude = 16 feet; find the area of rectangle.

21. How long a fence will it take to inclose a lot 30 feet long and 20 feet wide, and how many square feet will there be in the lot?

22. How many square yards has a rug 12 feet by 9 feet, and how many yards of binding are required for it?

23. How many feet of molding will be needed for a door frame 8 feet high and 4 feet wide?

QUERY. — On how many sides is the molding placed?

24. If the base of a rectangle is 8 inches, how many rows of square inches must it have to contain 24 square inches? What is its altitude?

25. If the base of a rectangle is 9 inches, and its area is 36 square inches, how many rows of squares has it?

26. If the base of a rectangle is 8 inches, and its area 32 square inches, what is its altitude?

27. Base of a rectangle = 9 inches; area = 63 square inches; required the altitude.

28. Base of a rectangle = 10 centimeters; area = 50 square centimeters; find the altitude.

29. Base = 10 inches; area = 35 square inches; required the altitude.

30. Base = $12\frac{1}{2}$ inches; area = 50 square inches; required the altitude.

31. Base = $7\frac{1}{2}$ inches; area = 60 square inches; required the altitude.

32. Base = $4\frac{1}{2}$ inches; area = $94\frac{1}{2}$ square inches; required the altitude.

33. Base = 8 inches; area = 72 square inches; required the altitude and perimeter.

34. Base = 9 centimeters; area = 45 square centimeter; find the altitude and perimeter.

35. How many millimeters are there in the perimeter of a rectangle whose base is 21 millimeters and area 105 square millimeters?

36. How many millimeters are there in the perimeter of a rectangle whose base is 3 centimeters and whose area is 6 square centimeters?

37. How many yards of binding are required for an oilcloth which is 10 feet long and contains 90 square feet?

38. How many yards of binding are required for a rug 15 feet long which contains 20 square yards?

39. How many yards of fringe are required for the ends of a pair of curtains, each of which is 10 feet long, and contains 50 square feet?

40. If the altitude of a rectangle is 3 inches, and it contains 30 square inches, how many square inches must there be in each row? What is the base of the rectangle?

41. If the area of a rectangle is 16 square inches, and its altitude is 2 inches, how many square inches are there in each row?

42. If the altitude of a rectangle is 7 inches, and its area is 56 square inches, what is its base?

43. Altitude of rectangle = 8 inches; area = 96 square inches; find base.

44. Altitude of rectangle = $9\frac{1}{2}$ inches; area = 95 square inches; find base.

45. Altitude of rectangle = $6\frac{1}{4}$ inches; area = 50 square inches; find base.

46. Altitude of rectangle = $4\frac{1}{6}$ inches; area = $83\frac{1}{3}$ square inches; find base.

47. Altitude of rectangle = 8.5 inches; area = 170 square inches; find base.

48. Altitude of rectangle = 7.5 inches; area = 87.5 square inches; find base.

49. Altitude of rectangle = 24 millimeters; area = 288 square millimeters; find base.

50. Altitude of rectangle = $3\frac{1}{2}$ centimeters; area = 77 square centimeters; find base.

51. How many centimeters are there in the perimeter of a rectangle whose altitude is 15 centimeters and area 225 square centimeters?

52. How many millimeters are there in the perimeter of a rectangle whose area is 15 square centimeters and whose altitude is 5 centimeters?

53. If the altitude of a rectangle is 9 inches and its area is 81 square inches, how long is the base? What kind of a rectangle is it?

54. What is one side of a square whose area is 9 square inches?

55. Find the perimeter and the area of a square whose side is 5 inches.

56. Find the perimeter of a square whose area is 36 square inches; of one whose area is 49 square inches; 64 square inches; 81 square inches; 100 square inches.

57. How many square inches are there in two rectangles, the first being 20 inches long and 10 inches wide, the other 19 inches long and 11 inches wide?

58. How many square inches are there in three squares, the side of the first being 4 inches, of the second 6 inches, of the third 8 inches?

59. How many square inches are there in two squares, the perimeter of the first being 72 inches, and that of the second 40 inches?

60. Find the combined length of the perimeters of two squares, one of which contains 36 square inches, and the other 25 square inches.

61. How many rods of fencing will be required to inclose two square fields, one of which contains 9 square rods, the other 49 square rods?

62. Draw a figure which is the sum of two squares placed so that they have a common side, each square being 7 inches in length and breadth. Find its area and perimeter.

63. Draw a figure which is the sum of two squares placed together on a line, one 5 inches, the other 4 inches in dimensions, and find its area and perimeter.

64. Draw a figure which is the sum of three squares placed together on a line, one 6 inches, another 5 inches, the other 4 inches in dimensions, and find its area and perimeter.

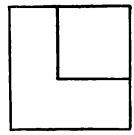
65. Find the difference in the area of two rectangles, one 13 inches long and 11 inches wide, and the other 11 inches long and 5 inches wide.

66. Find the difference in the area of two squares whose sides are respectively 7 inches and 5 inches.

67. Find the difference in area between two squares, the perimeter of the greater being 32 inches, and that of the less being 28 inches.

68. Find the difference in the number of rods of fence required to inclose two square fields, one containing 64 square rods, the other 36 square rods.

69. Cut a 4-inch square from the upper right-hand corner of a 7-inch square, and find the area and perimeter of the figure remaining.

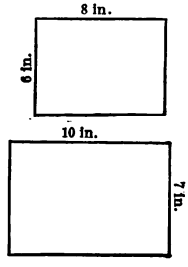


70. Draw a figure which is the difference of two squares, one 6 inches and the other 4 inches in dimensions, the smaller square being cut from the right-hand lower corner of the larger, and find its area and perimeter.

71. Draw a figure which is the difference of two squares, one 8 inches and the other 5 inches in dimensions, the smaller being subtracted at the left-hand lower corner from the larger, and find its area and perimeter.

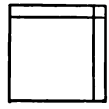
72. Place a square 3 inches in dimensions in the upper left-hand corner of one 10 inches in dimensions, and find the area and perimeter of the figure which shows their difference.

73. Give the length, width, and area of the rectangle which must be cut off from the given rectangle that the largest possible square may remain.



74. Give the dimensions and area of the rectangle which, added to one side of the given rectangle, will make it a square.

75. Give the number of square inches in the two rectangles and little square which, cut off from the given 8-inch square, will reduce it to a 7-inch square.



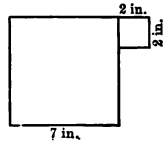
76. Find the number of square inches in the two rectangles and little square which, added to a 5-inch square, will make it a 7-inch square.

77. If a 5-inch square is cut off from the right-hand upper corner of a 10-inch square, how many square inches will there be in the irregular figure remaining? Find its perimeter.

78. If a 4-inch square is cut from the lower left-hand corner of a 9-inch square, what is the area and what is the perimeter of the remaining figure?

79. Add to a 5-inch square the two rectangles and small square that will make it a 6-inch square, and find the area of the additions.

80. If a 2-inch square is added at the right-hand upper corner of a 7-inch square, what is the area of the irregular figure thus formed? How long is its perimeter? How many sides has it?



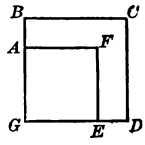
NOTE. — A plane figure bounded by six sides is called a **Hexagon.** The figure in Ex. 80 represents a certain kind of hexagon.

81. Draw several differently shaped figures each having six sides and write upon them the name which belongs to them

82. Find the area of the figure which is the sum of two squares, respectively, 9 inches and 6 inches in dimensions, so placed that a side of one square is a continuation

of a side of the other, and an angle of one square is adjacent to an angle of the other. Find its perimeter.

83. How many square inches are there in the figure (represented by *ABCDEF*) which is the difference of the square *GDCB*, 10 inches each way, and *GEFA*, 7 inches each way?



84. Draw the square of the line *AB*, 5 inches long.

NOTE. — The square of a line is the square of which the line forms a side.

85. Draw the rectangle of the line, *AB* and *BC*, *AB* being 8 inches, *BC*, 7 inches.

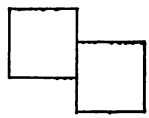
NOTE. — The rectangle of two lines is the right-angled, four-sided plane figure formed by using one of the lines as the measure of length and the other as the measure of width. It is also called the product of the lines.

86. Draw a figure which is the sum of the squares of two lines, each 6 inches, and determine the number of square inches in it.

87. Place the squares of two lines, each 7 inches, so that they shall have one common side, and find the area and perimeter of the rectangle thus formed.

88. Find the perimeter of the rectangle which is the sum of the squares of two lines, each 5 inches, so placed that they have a common side.

89. Place the squares of two lines, each 8 inches, so that the vertex of one of the angles of each square shall coincide with the middle point of a side of the other, and find the perimeter of the figure thus formed. How many sides has the figure?

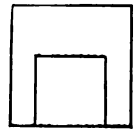


90. Place the squares of two lines, each 10 inches, so that one half of a side of each square shall coincide with one half of a side of the other, and find the perimeter of the figure thus formed.

91. Find the area and perimeter of the figure formed by subtracting the square of a 2-inch line from the upper left-hand corner of the square of a 7-inch line.

92. Find the area and the perimeter of the figure which is the difference of the squares of the lines *AB*, 8 inches, and *AC*, 6 inches, the subtraction being made from the upper right-hand corner.

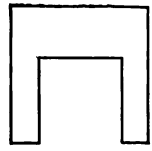
93. Place the square of the line *AB*, 6 inches, inside the square of the line *CD*, 10 inches, so that the middle points of their bases shall coincide.



How many square inches are there in the figure which is the difference of their squares? How long is its perimeter? How many sides has it?

94. Find the area and perimeter of the figure which is the difference of the squares of the lines *AB*, 9 inches, and *CD*, 7 inches, placed as in Ex. 93.

95. A mantel 40 inches high and equally wide is set with a grate 23 inches high and wide. How many square inches are there in the mantel?

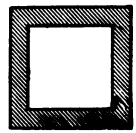


96. Place the small square in Ex. 93, one inch farther to the left, and find the area and the perimeter of the figure which is the difference of the squares.

97. Place the same squares so that their middle points coincide. How many square inches are there in their difference?

98. Cut a square 3 inches in dimensions from the middle of a square 7 inches in dimensions, and tell how many square inches are left.

99. The frame of a looking-glass is 30 inches in length and width; the glass is 20 inches square. How many square inches are there in the frame?



100. A garden 40 feet square gives so much space to a walk along its edge that the remaining space is 28 feet square. How many square feet are there in the walk, and what is its width?

101. The perimeter of a certain square is 4 inches longer than that of a smaller square, and the sum of their perimeters is 36 inches. How many square inches are there in the sum of the squares?

SUGGESTION. — Let x = perimeter of smaller square.

102. The perimeter of a square is 8 inches longer than that of a smaller square, and the sum of their perimeters is 56 inches. How many square inches are there in the difference of their squares?

103. Draw a diagram of a rectangular lot 7 rods long and 5 rods wide, representing each rod by a half inch, and show its area.

104. Draw a diagram of a room 7 feet long and 5 feet wide, representing each foot by a centimeter, and find the area of the floor.

105. Draw a diagram of a rug 12 feet long and 9 feet wide on a scale of 1 centimeter to the foot.

106. The bottom and each of the sides of a drawer 12 inches long, 10 inches wide, and 4 inches deep, are in the

shape of what geometrical figure? How many square inches of blue velvet would it take to line the drawer?

107. How many square inches are there in all the surfaces of a box 8 inches long, 6 inches wide, 4 inches high? How many pairs of equal rectangular faces has the box?

108. How many square inches are there in all the surfaces of a brick 8 inches long, 4 inches wide, and 2 inches thick? How many pairs of equal rectangular faces has the brick?

NOTE. — A solid which has six rectangular faces, of which the opposite ones are equal and parallel, is called a **Right Prism** or **Parallelopiped.**

109. Is the space inside the drawer mentioned in Ex. 106 a geometric or a physical solid?

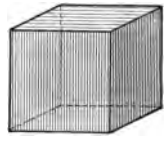
NOTE. — Any material object is a physical solid, as a book, or box, or house. The space occupied by a material object forms a geometric solid.

110. Think of a chalk box and imagine the geometric solid which corresponds to it. What name is given to that geometric solid?

111. Were you ever inside of a parallelopiped?

112. How many square feet are there in the walls, ceiling, and floor of a room 10 feet long, 9 feet wide, 8 feet high?

113. What name is given to those prisms whose faces are all squares?



114. How many square inches are there in all the faces of a cube, one edge of which is 5 inches?

115. How many inches are there in all the lines formed by the meeting of two surfaces of the same cube?

116. How many angles are formed on each face of a cube by its boundary lines?

117. Find the number of degrees in the sum of all the angles formed on the faces of a cube by their boundary lines.

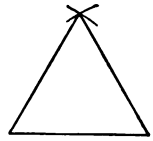
118. How many square inches are there in the surfaces of a box whose height is 4 inches, its width twice its height, and its length 3 times its width?

119. How many square inches are there in the surfaces of a brick which is 9 inches long, $\frac{1}{2}$ as wide as long, and $\frac{1}{2}$ as thick as wide?

CHAPTER VI.

TRIANGLES AND LINES.

1. Draw a line 5 inches long. With one end of the line as a center and a radius of 5 inches, describe an arc. With the other end as a center and the same radius, describe an arc intersecting the first. Join the point of intersection with the ends of the line. What kind of a plane figure is thus formed?



NOTE. — A triangle whose sides are all equal is called an **Equilateral Triangle.**

2. How many centimeters are there in the perimeter of an equilateral triangle, one side of which is 8 millimeters?

3. Find the length in decimeters of one side of an equilateral triangle whose perimeter is 48 centimeters.

4. Find the difference between the length of the perimeter of an equilateral triangle each of whose sides is 7 inches, and that of an equilateral rectangle each of whose sides is 7 inches.

QUERY. — What does "equilateral" mean?

5. The perimeter of a triangle whose sides are each 6 centimeters is what fractional part of the perimeter of a hexagon each of whose sides is 6 centimeters?

6. Draw two equal lines making any angle, and join their extremities. What plane figure is formed?

NOTE. — A triangle which has two of its sides equal is called an **Isosceles Triangle.** The unequal side is called the **Base.**

7. Find the base of an isosceles triangle whose perimeter is 90 millimeters and each of whose equal sides is 35 millimeters.

8. Find each of the equal sides of an isosceles triangle whose perimeter is 40 inches and base 10 inches.

9. Each of the equal sides of an isosceles triangle is double the base, and the perimeter is 45 centimeters. Find each side.

SUGGESTION. — Let x = base.

10. The base of an isosceles triangle is 5 inches longer than each of its equal sides, and the perimeter is 35 inches. Find each side.

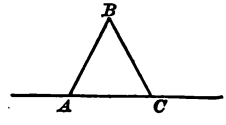
QUERY. — What shall x equal?

11. The sum of the equal sides of an isosceles triangle is 4 times the base, and the perimeter is 15 inches. Find each side.

12. Cut out an isosceles triangle, fold it so that the equal sides shall coincide, and show the truth of the following theorem:

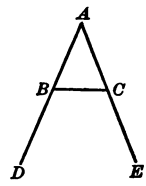
PRINCIPLE 17. — *In an isosceles triangle the angles opposite the equal sides are equal.*

13. In the isosceles triangle ABC the angle BAC is 70°. How many degrees are there in each exterior angle formed by prolonging the side AC? Quote principle.



NOTE. — An angle formed by prolonging one side of a polygon is called an **Exterior Angle.**

14. The exterior angle DBC, formed by prolonging one leg of the isosceles triangle ABC, is 115°. Find each of the base angles and the exterior angle BCE.



15. The exterior angle formed by prolonging the base of an isosceles triangle contains 3 times as many degrees as the interior base angle. How many degrees are there in the sum of the base angles? How many in their difference?

16. The exterior angle formed by prolonging one of the legs of an isosceles triangle is 26° more than a base angle. How many degrees are there in each base angle?

17. Draw and cut out an equilateral triangle. Fold it so that two of its equal sides shall coincide. Are the angles opposite them equal? Smooth it out and fold it so that another pair of equal sides shall coincide. Are the angles all equal?

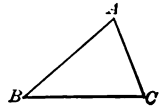
NOTE. — A triangle whose angles are all equal is **Equiangular.**

PRINCIPLE 18. — *An equilateral triangle is equiangular.*

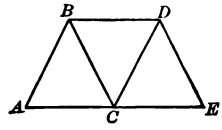
18. In the equilateral triangle ABC, of which BC is the base, which is the greater, the vertical angle or one of the base angles?

NOTE. — Any side of a triangle on which it may be supposed to stand may be called the **Base,** and the angle opposite the base is called the **Vertical Angle.**

19. If AB is the base of the triangle ABC, what is the vertical angle? If B is the vertical angle, what is the base?



20. Draw three equilateral triangles of the same dimensions and place them so that they have the vertex of an angle of each at a common point, as at C in the figure. It will be seen that a straight line is formed by the bases of the outer triangles.



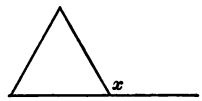
The straight line ACE is what fractional part of the broken line $ABDE$?

21. How many degrees are there in the sum of the angles at C? Quote geometric principles.

22. How many degrees are there in each of the angles at C? Give reasons.

23. How many degrees are there in angle BAC? In angle ABC? How many are there in each angle of each equilateral triangle? How many in the sum of the angles of either one of the equilateral triangles?

24. How many degrees are there in the angle x, formed by prolonging a side of an equilateral triangle?



25. Draw an equilateral triangle and prolong its sides so as to form an exterior angle at each vertex. Cut out the exterior angles and place them around a common point. Will they form a continuous surface around the point? How many degrees are there in their sum?

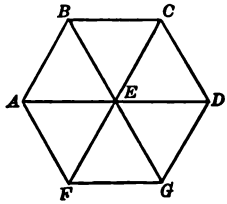
26. Draw a regular polygon of three sides and name it.

NOTE. — When a polygon is equilateral and equiangular it is called a **Regular Polygon.**

27. Draw a regular polygon of four sides and write its name upon it. How many right angles has it? How many degrees are there in all its angles?

28. Cut out four equal squares and place them around a common point so that an angle of each shall have its vertex at the point. Will they form a continuous surface around the point?

29. Make six equal equilateral triangles and place them around a common point, as E. They form a regular hexagon. How many degrees are there in each angle formed at E? How many degrees are there in angle ABC? BCD? In each angle of the hexagon? Give reasons.



30. How many sides has the polygon $ADCB$? Which of its sides are parallel?

NOTE. — A quadrilateral which has only two of its sides parallel is called a **Trapezoid.**

31. Draw a trapezoid and show into how many triangles it can be divided by one line.

32. If each of the sides of the equilateral triangles in Ex. 29 is 8 centimeters, what is the perimeter of the trapezoid $BGDC$?

33. Inclose a surface by three unequal lines, and name the plane figure thus formed.

NOTE. — A triangle whose sides are all unequal is called a **Scalene Triangle.**

34. Construct a triangle whose sides are 5 inches, 6 inches, and 7 inches, using the 7-inch line as the base.

SUGGESTION. — With the extremities of the base as centers describe arcs, with radii respectively 5 inches and 6 inches, and join the point of intersection with the extremities of the base.

35. Construct a triangle whose sides are 8 centimeters, 7 centimeters, and 6 centimeters.

36. Construct a triangle whose sides are 9 inches, 3 inches, and 4 inches. Explain.

37. What is the perimeter of the scalene triangle ABC, in which AB is 12 inches, BC is 2 inches longer than AB, and AC is 3 inches longer than BC?

38. The perimeter of a scalene triangle is 47 inches; one side is 11 inches and another side is $1\frac{1}{2}$ times as long. Find the third side.

39. The triangle ABC, whose perimeter is 54 inches, has the side AB 7 inches longer than the side BC, and the side BC 10 inches longer than the side AC. Find each side.

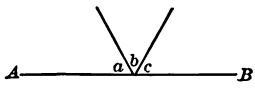
SUGGESTION. — Let $x = AC$.

40. The scalene triangle ABC has the side AB 12 inches longer than the side AC, and the side AC 8 inches longer than the side BC. The perimeter is 73 inches. Find each side.

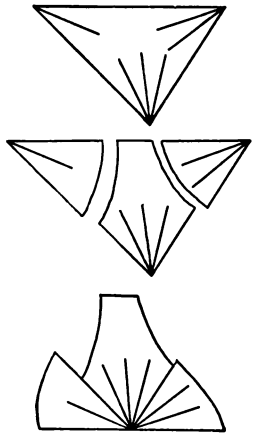
41. The side XY of the scalene triangle XYZ is 11 inches longer than the side YZ, and the side XZ is 17 inches longer than the side YZ. The perimeter is 88 inches. Find each side.

42. The side DE of the scalene triangle DEF, the perimeter of which is 65 inches, lacks 8 inches of being twice as long as the side EF. The side DF lacks 17 inches of being 3 times as long as EF. Find the length of each side.

43. Quote the geometric principle which tells how many degrees there are in the sum of the angles a, b, and c, AB being a straight line.



44. Draw and cut out a triangle, dividing each of its angles by straight lines. Cut off two of the corners. Place them beside the third corner.



It will be seen that all the angles will have their vertex at a common point, and will lie on the same side of a straight line. How many degrees are there in the sum of all the angles?

PRINCIPLE 19. — *The sum of the angles of any triangle is equal to two right angles, or 180 degrees.*

45. Draw different kinds of triangles, and repeat the process given in Ex. 44 illustrating Prin. 19.

46. How many degrees are there in the vertical angle of an isosceles triangle whose base angles are each 80°?

47. How many degrees are there in each base angle of an isosceles triangle whose vertical angle is 50°?

48. The angle a of a triangle is 80°, and angle b is 3 times angle c. Find angle b and angle c. Let $x = ?$

49. Make a right angle, and join the extremities of the lines. How many degrees are there in the sum of the two angles that are not right angles?

NOTE. — A triangle which has a right angle is a **Right Triangle.** The side opposite the right angle is called the **Hypotenuse.**

50. How many degrees are there in each angle of an isosceles triangle whose vertical angle is equal to the sum of the base angles?

51. Draw a right-angled isosceles triangle having its equal sides each 5 inches long, and show how many degrees there are in each of the complementary angles.

QUERY. — When is one angle complementary to another?

52. Draw a right-angled isosceles triangle having its equal sides each 10 inches long, and show how many degrees there are in each of the complementary angles.

53. In the scalene triangle ABC the angle A is a right angle, and the angle B is 4 times the angle C. How many degrees are there in each of the complementary angles?

54. Find each of the complementary angles in a right triangle, in which one acute angle is 5 times the other.

55. Construct an angle of 60° at one extremity of a line and one of 70° at the other extremity. Prolong the lines until they meet. How many degrees are there in the third angle?

NOTE. — A triangle in which all the angles are acute is an **Acute-angled Triangle.**

56. Draw an acute-angled triangle in which one angle is 80°.

57. Draw an obtuse angle, and join the ends of the lines which form it. What kind of triangle is thus formed?

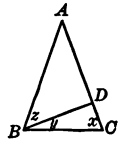
NOTE. — A triangle which has an obtuse angle is an **Obtuse-angled Triangle.**

58. Can you draw an acute-angled triangle in which the sum of any two angles is less than a right angle?

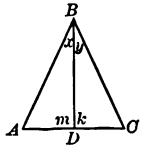
59. To what is the sum of the oblique angles of a right triangle equal?

60. Try to draw an obtuse-angled triangle in which the sum of the acute angles is greater than a right angle. Explain.

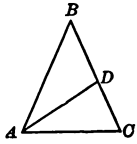
61. BD is perpendicular to AC, one of the legs of the isosceles triangle ABC, whose vertical angle is 40°. How many degrees are there in each of the angles x, y, and z?



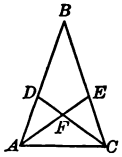
62. BD bisects the vertical angle of the isosceles triangle ABC, a base angle of which is 65°. How many degrees are there in angle x? In angle y? In angle m? In angle k? What is the relative position of BD and AC?



63. AD is a bisector of a base angle of the isosceles triangle whose vertical angle B is 48°. How many degrees are there in angle ADB?



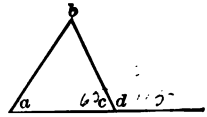
64. In the isosceles triangle ABC the vertical angle is 38°. AE bisects one base angle. DC bisects the other. Angles of how many degrees are formed at F?



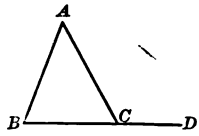
65. Angles of how many degrees are formed by the intersection of the bisector of the vertical angle and the bisector of a base angle in the triangle whose base angles are each 20°?

66. What angles are formed by the intersection of the lines drawn from the vertices of the base angles perpendicular to the legs of an isosceles triangle whose vertical angle is 30°?

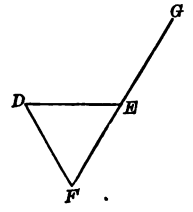
67. Angle $a = 55°$. Angle $b = 60°$. Angle $c = ?$ Angle $d = ?$ Which is the greater, $a + b$, or d?



68. Exterior angle ACD = twice angle ACB; $ABC = 70°$; find angle BAC. Which is the greater, the exterior angle, or the sum of the two interior angles which are opposite?



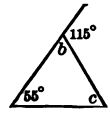
69. DEF is an equilateral triangle. How many degrees has the exterior angle DEG? Compare the number of degrees in the exterior angle with the number of degrees in the opposite interior angles.



70. Draw an isosceles triangle whose vertical angle is 30°, and compare the number of degrees in any exterior angle with the number of degrees in the opposite interior angles. Can you give a reason for the fact stated in the following theorem?

PRINCIPLE 20.—*An exterior angle of a triangle is equal to the sum of the opposite interior angles.*

71. Find angle c and angle b.

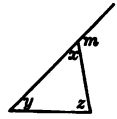


72. Every degree is divided into 60 equal parts, called minutes, marked (′). If one angle of a triangle is 37° 30′, and another is 50° 30′, how many degrees are there in the angle exterior to the third angle?

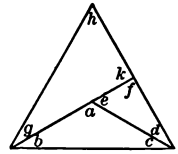
73. How many degrees are there in the exterior vertical angle of a triangle, each of whose base angles is 27° 15′?

74. How many degrees are there in the exterior vertical angle of an isosceles triangle, each of whose exterior base angles is 110°?

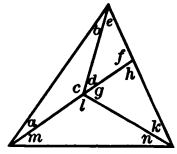
75. Angle $x=55°$. Angle $z=?$
Angle $y=48°$. Angle $m=?$



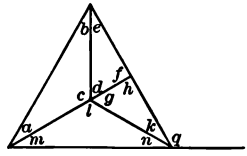
76. What angles are exterior to the triangle whose angles are e, f, and d? Name the angle exterior to the triangle whose angles are g, h, and k? If $a=120°$, $c=20°$, $f=91°$, and $h=65°$, how many degrees are there in each angle of the three triangles?



77. Given, angle $m=35°$, angle $n=30°$, angle $e=40°$, angle $d=40°$, angle $b=20°$.
Required, the angles l, g, h, k, c, a, f.

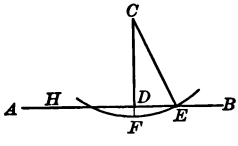


78. What angle is exterior to the triangle whose angles are e, d, and f? g, h, and k? a, b, and c? m, l, and n?



79. Try to draw an isosceles triangle having an exterior base angle of 75°. Explain.

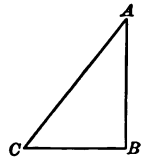
80. Let AB be a given line and C a point above it. Draw the perpendicular CD and the oblique line CE. With CE as a radius and C as a center, draw an arc cutting the line AB. Continue the line CD till it touches the arc in a point which we call F. Which is longer, CD or CF? Compare CF and CE. Compare CD and CE.



81. Take some other point in the line AB, as the point H, and show that an oblique line drawn to it from the point C is greater than the perpendicular drawn from the same point to the same line.

PRINCIPLE 21. — *If from a point without a line a perpendicular and oblique lines are drawn to that line, the perpendicular is shorter than any oblique line.*

82. Which is the longest side of a right triangle? Give reasons. In the triangle right-angled at B, what line is the shortest distance from the point A to the line of the base CB?

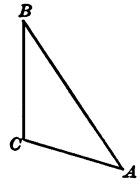


83. Draw an acute-angled triangle ABC and a dotted line to show the shortest distance from the point A to the line BC.

NOTE. — A perpendicular drawn from the vertical angle of a triangle to its base or its base prolonged is called the **Altitude** of the triangle.

84. Reproduce the triangle in Ex. 83 and draw the altitude from the point C. From the point B.

85. Draw a dotted line to show the altitude from the point B to the base AC prolonged; the altitude from the point C to the base AB; the altitude from the point A to the base BC prolonged.



86. Draw a square 8 inches each way and its diagonal. What kind of triangles are formed? Show the base and altitude of each. What part of the square is each? What is the area of each?

87. Draw a rectangle 8 inches by 6 inches and its diagonal. What kind of triangles are formed, and what is the area of each?

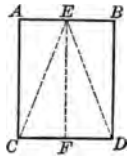
88. Is it true that the area of a right triangle is equal to one half that of the rectangle having the same base and altitude, or that it equals one half the product of its base by its altitude? Illustrate your answer.

89. What is the area of a right triangle whose base is 17 feet and altitude 9 feet?

90. Find the area of a right triangle having a base of $3\frac{1}{2}$ feet and an altitude of 16 inches.

91. Find the area of a right triangle having a base of $7\frac{1}{4}$ feet and an altitude of $3\frac{1}{3}$ feet.

92. Reproduce the rectangle *ABDC*, making *AB* 6 inches in length and *AC* 8 inches. Join *E*, the middle point of *AB*, with *C* and *D*. What kind of a triangle is *CED*?



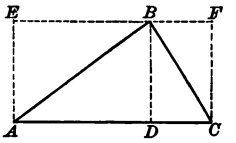
93. Draw the dotted line *EF* perpendicular to *CD*. What is the altitude of the triangle *CED*? How long is *EF*? What part of the rectangle *EBDF* is the triangle *FED*? The triangle *CEF* is what part of the rectangle *AEFC*? The triangle *CED* is what part of the rectangle *ABDC*? What is the area of the triangle *CED*?

NOTE. — Dotted lines added to a figure to help in studying it are called **Construction Lines.**

94. Draw an isosceles triangle and a construction line showing its altitude. Draw construction lines forming

about the triangle a rectangle having the same base and altitude. If the base is 7 inches and the altitude 10 inches, what is the area of the triangle?

95. Reproduce the scalene triangle *ABC*, drawing the construction line *BD* to represent its altitude. Construct the rectangle *AEFC* about the triangle.



The triangle *DBC* is what part of rectangle *BDCF*?

The triangle *ABD* is what part of rectangle *EBDA*?

The triangle *ABC* is what part of rectangle *AEFC*?

NOTE. — You will observe that we find the areas of all these different kinds of triangles in the same way, by taking half the product of the base and altitude.

PRINCIPLE 22. — *The area of a triangle is equal to one half the product of its base and altitude.*

96. Draw a triangle and show in what way you find its area.

97. Find the area of a triangle whose base is 15 feet and altitude 10 feet.

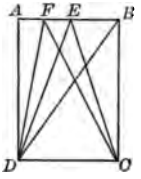
98. The area of a triangle is 30 square feet; its base, 6 feet; find its altitude.

99. The area of a triangle is 40 square feet; its base, 10 feet; find the altitude.

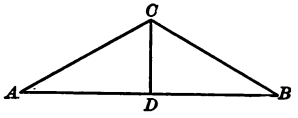
100. The area of a triangle is 48 square feet; the altitude, 12 feet; find the base.

101. The area of a triangle is 56 square feet; the altitude, 8 feet; find the base.

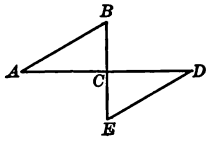
102. Draw a rectangle $ABCD$ 8 inches by 6 inches. Join the middle point of the upper base with the extremities of the lower base DC, making the triangle DEC. Draw a diagonal, making the triangle DBC. Join F, a point on the upper base, with the extremities of the lower base, making the triangle DFC. What kind of a triangle is each and which is the greatest? Give reasons.



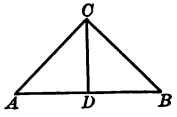
103. CD, 4 inches, is perpendicular to AB, 14 inches, at its middle point. Find the area of each triangle.



104. C is the middle point of AD, 22 inches, and of BE, 12 inches. The angles at C are right angles. Find the area of each triangle.

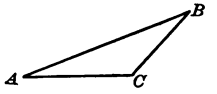


105. AB, 10 inches, is met perpendicularly at its middle point by CD, 5 inches. Find the area of each of the triangles thus formed. What kind of triangles are they?

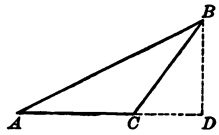


106. What is the altitude of a triangle? Does the line representing the altitude always fall inside the triangle?

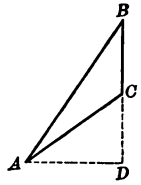
107. Draw a line which represents the altitude of the triangle ABC when AB is considered the base; when AC is considered the base; when BC is considered the base.



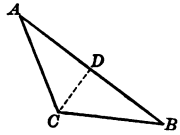
108. *BD*, a perpendicular to the line *AC*, striking its prolongation, is 4 inches; *AC* is 5 inches; find the area of *ABC*.



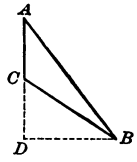
109. *BC* = 8 inches; *AD* perpendicular to *BC* prolonged = 9 inches; find the area of the triangle.



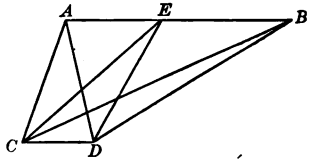
110. *AB* = 12 inches; *CD* perpendicular = 4 inches; find the area of the triangle.



111. The distance *BD* from the point *B* to *AC* prolonged = 8 inches; *AC* = 5 inches; find the area of triangle *ACB*.



112. *AB* and *CD* are parallel. Which of the three triangles *CAD*, *CED*, or *CBD*, is the greatest?



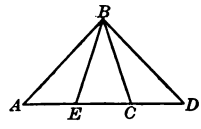
113. Draw an isosceles triangle *ABC* and two right triangles having the same base and equal area. Draw two obtuse-angled triangles having the same base and equal area.

114. Draw the line *AB* 8 inches. With the point *A* as a center with a radius of 6 inches describe an arc. With the point *B* as a center with a radius of 7 inches draw an

arc intersecting the first. Join C, the point of intersection, with A and B, making the scalene triangle ABC. Construct another triangle with the base 7 inches, and the other sides 8 inches and 6 inches. Construct a third triangle with the base 6 inches, and the other sides 8 inches and 7 inches. Cut out the three triangles and superpose one upon another, and show that they may be made to coincide in all their parts.

PRINCIPLE 23. — *Two triangles are equal when the three sides of the one are respectively equal to the three sides of the other.*

115. AB and BD are each 7 inches, BE and BC each 5 inches, AE, EC, and CD each 3 inches. Compare the perimeters and areas of the triangles ABE and CBD. Of ABC and EBD.



116. With any point as a center and any radius, describe an arc. Choosing a point in the arc, mark with the dividers two other points in the arc equally distant from the chosen point, and connect them with it. Connect the three points with the center of the circle, and show that the triangles thus formed are equal.

117. Join the vertex of an isosceles triangle with the middle point of the base, and show that the triangles thus formed are equal in all their parts.

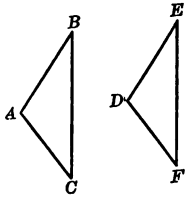
118. With a given point as a center and any radius, draw an arc. Connect any two points in the arc by a chord, and draw radii to its extremities. What kind of a triangle is thus formed? Join the middle point of the chord with the center. Show that the two triangles thus formed are equal in all their parts.

119. Construct the angle ABC 50°, making BA 5 inches and BC 7 inches. Join AC. Construct the equal angle DEF with lines respectively equal. Join DF. Superpose angle DEF upon angle ABC so that the 5-inch lines shall coincide and the 7-inch lines shall coincide. Since point D is on point A, and point F is on point C, must not the lines DF and AC coincide? Are the triangles equal in all their parts? Measure the angles, and show that the corresponding angles are equal.

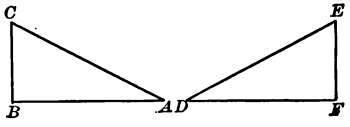
NOTE. — The corresponding angles of a figure are called **Homologous Angles,** and the corresponding sides **Homologous Sides.**

PRINCIPLE 24. — *If two triangles have two sides and the included angle of one respectively equal to two sides and the included angle of the other, the triangles are equal in all their parts.*

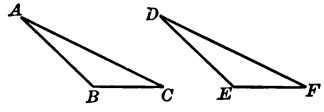
120. AB and DE are each 8 centimeters; AC and DF are each 7 centimeters. The angles A and D are equal. The perimeter of the triangle ABC is 27 centimeters. Find the side EF.



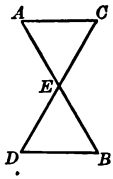
121. AB and DF are each 8 inches in length; AC and DE each 10 inches; EF is 6 inches. The angles A and D are each 30°. How long is the perimeter of ABC?



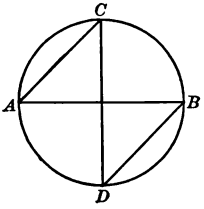
122. Angle A = angle D. $AB = DE$; $AC = DF$; $DE = EF + 4$ inches; $DF = DE + 7$ inches; $DE + EF + DF = 42$ inches; required BC.



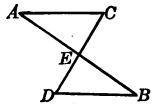
123. The lines AB and CD are each 12 inches long, and bisect each other at E. The angle DEB is 60°; the side AC is 6 inches. How long is DB? By what geometric principles can you prove it



124. Draw a circle and two diameters AB and CD, crossing at an angle of 90°. Join AC and DB. Can you prove that the triangles thus formed are equal?



125. $AB = 12$ inches; $CD = 8$ inches; E is the middle point of each line. If AC is 7 inches, what is the perimeter of DEB? Quote principle.



126. Draw the horizontal line AB 6 inches, and at its middle point C erect a perpendicular CD 4 inches. Join AD and DB. What line is common to both triangles? If AD is 5 inches, what is the sum of the perimeters of the triangles?

127. Draw the vertical line AB 10 millimeters and mark its middle point C. Draw the perpendicular line CD extending 12 millimeters to the right. Join AD and BD. BD being 13 millimeters, what is the sum of the perimeters of the triangles thus formed?

128. Draw a circle with a radius of 8 centimeters, and at its center make two angles of 120° each. Draw the chords of the arcs which measure these angles and show that the two triangles thus formed are equal.

129. Draw the chord of the remaining central angle and compare the triangle thus formed with either of the

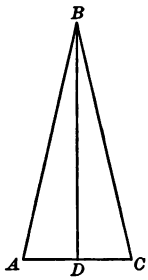
others. What kind of a triangle is formed by the three chords?

130. Show how you inscribe an equilateral triangle in a circle.

NOTE. — A polygon is said to be inscribed in a circle when the vertices of all the angles of the polygon are in the circumference.

131. Include an angle of 40° by two lines, each 6 inches long, and join their extremities. What kind of a triangle is thus formed? Divide the included angle into two angles of 20° each, and continue the construction line to the opposite side or base. Show that the two triangles thus formed are equal in all their parts.

132. In the isosceles triangle ABC the angle B is bisected by BD. What two sides and included angle of the right-hand triangle equal two sides and the included angle of the other triangle? Quote the principle that declares the equality of these triangles. If the angle A is 75°, how many degrees is the angle C? Since the two angles at D are equal, how many degrees are there in each? What kind of a line is BD? If AC is 10 inches, how long is AD?



Show the truth of the following principle:

PRINCIPLE 25. — *In an isosceles triangle the line which bisects the vertical angle bisects the base and the triangle, and is perpendicular to the base.*

133. The equal sides of an isosceles triangle whose perimeter is 70 inches are each 5 inches longer than the base. A perpendicular is drawn from the vertex of

the triangle to the base. Find the distance from the foot of the perpendicular to an extremity of the base.

134. In an isosceles triangle whose perimeter is 55 inches the base lacks 5 inches of being as long as one of the equal sides. The triangle is divided by a perpendicular from the vertex to the base. Find the distance from the vertex of a base angle to the foot of the perpendicular.

135. In the isosceles triangle whose perimeter is 62 inches and whose equal sides are each 17 inches, what is the distance from the vertex of the base angle to the foot of a perpendicular from the vertical angle to the base?

136. In the equilateral triangle whose perimeter is 39 inches, how long is each segment of a side cut off by a perpendicular to that side drawn from the vertex of the opposite angle?

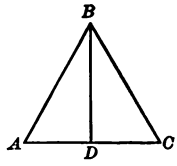
NOTE. — The word "segment" means "a piece cut off." A segment of a circle means a piece cut off by one line. One straight stroke through a sphere will cut off a segment of a sphere.

137. In the isosceles triangle ABC whose perimeter, which is 60 inches, is 37 inches longer than one of the equal sides, AC is the base and BD its bisecting perpendicular. Find AD and DC.

138. In the isosceles triangle ABC of which AC is the base, BC is 13 inches and the perpendicular BD, drawn from the vertex to the base, is 12 inches. The perimeter of each of the triangles into which it is bisected by the line BD is 30 inches. Find the length of the base.

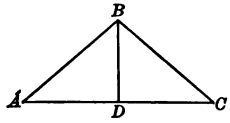
139. The sum of the equal sides of an isosceles triangle whose perimeter is 40 inches is 3 times the base. Find the distance from the foot of the perpendicular which bisects the base to the vertex of the base angle.

140. In the triangle ABC the lines BA and BC each meet the base 5 inches from the foot of the perpendicular BD. BA is 10 inches long; how long is BC? Quote the geometric principle.

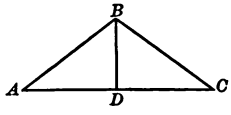


PRINCIPLE 26. — *If from a point without a line a perpendicular and oblique lines are drawn, two oblique lines which meet the given line at equal distances from the foot of the perpendicular are equal.*

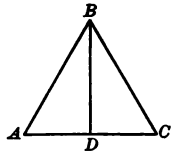
141. $AD = DC$; $AC = 16$ inches; BD is perpendicular to AC; $AB = 10$ inches; required the perimeter of triangle ABC.



142. BD, 6 inches, is perpendicular to AC, 16 inches, at its middle point; AB is 10 inches; find the sum of perimeters of triangles BDC and BDA.

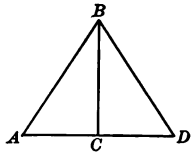


143. D is the middle point of AC; BD is perpendicular to AC. If BC is 12 inches and DC is 6 inches, is the triangle ABC scalene, isosceles, or equilateral? Prove.

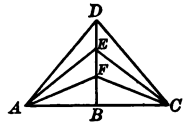


144. Draw an indefinite line, and at the point A in it erect a perpendicular AB 8 inches. Lay off AC 6 inches on the indefinite line, and on the other side of A lay off AD 6 inches. Compare DB and BC.

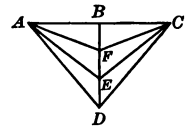
145. BC perpendicular to AD at its middle point is $\frac{4}{3}$ of CD and $\frac{4}{5}$ of BD. AC is 3 inches. What is the perimeter of the triangle CBD? Of the triangle ABC? Of the triangle ABD?



146. Can you show that the points D, E, and F in the line DB, drawn perpendicular to AC at its middle point, are each the same distance from A as from C? That $AE = EC$ and $AF = FC$?



147. Can you show that the points D, E, and F in the line DB, perpendicular to AC at its middle point, are each the same distance from A as from C?



148. Can you find a point in the line BD that is not just as far from A as it is from C?

PRINCIPLE 27. — *If a perpendicular is drawn at the middle point of a straight line, every point in the perpendicular is at the same distance from one extremity of that line as from the other.*

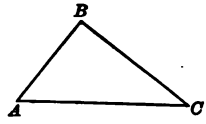
149. Draw the line AB. With the point A as a center, with a radius greater than half of AB describe an arc. With the point B as a center, with the same radius describe an arc intersecting the first arc above the line AB and also below it. Join the points of intersection. Can you see why the line which joins the intersections bisects the line AB?

150. Find the area of an isosceles triangle whose perimeter is 64 inches, one of whose equal sides is 25 inches, and whose altitude is 24 inches.

151. Find the approximate area of an equilateral triangle whose perimeter is 120 inches, and whose altitude is 34.6+ inches.

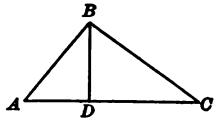
QUERY. — Why can we only find the approximate area of such a triangle?

152. The perimeter of the scalene triangle, right-angled at B, is 36 inches, BC is one foot in length and AB is $\frac{3}{4}$ of a foot. Find the area of the triangle.



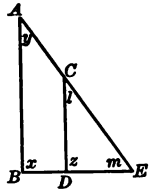
153. AB is perpendicular to BC; angle $BAD = 50°$; BD is perpendicular to AC.

How many degrees has each angle of the triangle ABD? Of triangle BDC? Of triangle ABC? What angle is common to triangle ABD and triangle ABC?



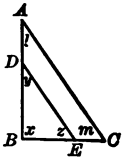
154. AB and CD are perpendicular to BE.

Angle $m = 55°$. Find the other angles of the triangles ABE and CDE. What angle is common to both triangles?

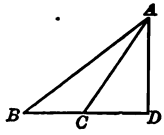


155. AB is perpendicular to BC; DE is parallel to AC.

Angle $l = 35°$. Find the other angles of the triangles ABC and DBE. What angle is common to both triangles?



156. AD is perpendicular to BD; $AD = 6$ inches; $BD = 8$ inches; $BC = CD$. Find the area of triangles ABC and ACD.



CHAPTER VII.

CUMULATIVE REVIEW NO. 2.

1. How many boundaries has a parallelopiped?

NOTE. — The boundaries of a solid are called **Surfaces.**

2. What forms the boundaries of a surface?

3. What are the extremities of lines?

4. A point is that which has position without length, breadth, or thickness. Did you ever see a point?

5. A line has length without breadth or thickness. How do we represent lines?

6. A surface has length and breadth without thickness. How many surfaces has a cube? How many has a sphere?

7. What name is given to those geometric forms that have length, breadth, and thickness?

NOTE. — A geometric form having length, breadth, and thickness is called a **Solid.**

QUERY. — Is the word "solid" used here in its ordinary sense?

8. If a point were moved in one direction through space, what would its path be?

9. What would be the path of a line which was moved through space in the direction of its length? What would be the path of a line moved in the direction perpendicular to its length?

10. Move a piece of paper through space in such a way that the path of one of its surfaces would be a geometric solid.

11. Cut out a square and lay it on the desk. If it were to rise in the air keeping itself always parallel to the desk, what geometric solid would it form when it had reached a height equal to one of its sides? What would you call it when it had reached a height greater or less than one of its sides?

12. If a circle should rise in the same way, what geometric solid would be formed?

NOTE. — Such a geometric solid is called a **Cylinder.**

13. Cut out a rectangle whose longer sides are each 22 inches and shorter sides each 8 inches. Place the shorter sides together in such a way that the longer sides form parallel circumferences. If circles were fitted to these circumferences, what kind of a solid would be formed? What would be the area of the curved surface? What is the length of the longest line that can be drawn on either of its plane surfaces?

14. In what geometric form are measures of bushels, half bushels and pecks generally made?

15. If the diameter of the bottom of a cylindrical tin pail is 10 inches, what is its circumference? If its height is 10 inches, how many square inches of tin were used in making its curved surface?

16. A straight line is one that does not change its direction at any point. What is a curved line?

17. Classify lines and define each class.

18. Classify angles and define each class.

19. What is the complement of an angle of 48° 50′?

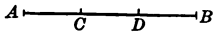
20. Work Ex. 19, substituting "supplement" for "complement."

21. How many degrees are there in angle x if its adjacent supplementary angle is 50° larger?

22. How many degrees are there in an angle whose complement is 10° more than 3 times as large?

23. How many degrees are there in each of the three angles formed on the same side of a straight line at a given point of that line if the first is 10° greater than the second, and the second 10° greater than the third?

24. $AB = 12$ inches; $AC = 4$ inches; $DB = AC$; Find CD.

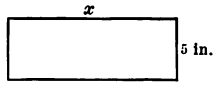


25. $CD = 6$ inches; $AB = 10$ inches; $AC = DB$; Find AC and DB.

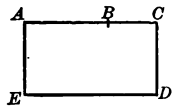
26. $AD = 12$ inches; $AC = 3$ inches; $AC = DB$; Find CB.

27. $AD = CB$; $CD = 7$ inches; $DB = 3$ inches; Find AB and AD.

28. If the area of the rectangle = 70 square inches, what is the value of x?



29. Given, $AE = 6$ inches; $AB = 7$ inches; area of rectangle = 66 square inches; required BC.



30. Find the area and perimeter of a rectangle each of whose long sides is 7 centimeters more than three times as long as a short side, and the sum of two adjacent sides of which is 27 centimeters.

31. Given a rectangle 12 millimeters long and 10 millimeters wide; find the area and perimeter of a rectangle drawn within it, having its sides parallel to the sides of the original rectangle and at a distance of 2 millimeters from them.

QUERY. — How much shorter is each side of the small rectangle than the homologous side of the large rectangle?

32. A rectangular garden 48 feet by 36 feet has a border 3 feet wide within its limits. How much surface is inside the border? How much is occupied by the border?

33. A square garden, whose dimensions are 8 yards, has a square flower bed in its center whose area is $\frac{1}{4}$ that of the garden, and whose sides are parallel to those of the garden. How long is one side of the flower bed, and how far is the middle point of one of its sides from the middle points of the parallel sides of the garden?

34. What is the sum of two lines 8 feet and 5 feet? Draw a figure on the scale of a centimeter to the foot showing the sum of the squares of those lines, and another showing the square of their sum.

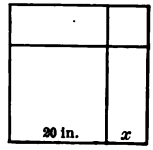
35. Find the area of the rectangles and small square which must be added to a 10-inch square to form a 15-inch square; to form an 18-inch square.

36. Given a 10-inch square to make into a larger square. Find by trial how wide the additions must be in order that the rectangles and small square shall contain 96 square inches.

37. Given a 10-inch square as a basis for a larger square. Find by trial how wide the additions must be that they may contain 69 square inches.

38. Given a 20-inch square to build a larger one. Find by trial the width of the necessary additions if they contain 176 square inches.

39. Find the length of x when the sum of the three additions to the 20-inch square is 384 square inches.



40. If x represents the side of a square, what represents the area of the square?

41. If a^2 represents the area of a square, what represents a side?

42. Find the area of a square whose perimeter is 100 inches.

43. Find the perimeter of a square whose area is 64 square inches.

44. Divide a semicircle into two sectors, one of which is 5 times the other. How many degrees are there in the angle of each?

45. The diameter of a circle is 42 inches. Find the length of the arcs of two sectors which compose the circle, the greater being 10 times the less. Illustrate.

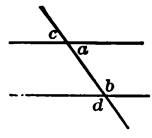
46. An arc which is a quadrant is $16\frac{1}{2}$ feet. Find the diameter.

47. What angle is the greatest common measure of a given angle and its complement, if the complement contains 24° more than the given angle?

SUGGESTION. — Let x = given angle.

48. What angle is the G. C. M. of a given angle and its supplement, if the supplement is 30° more than twice the given angle?

49. What angle is the G. C. M. of angle a and angle b, made by a transversal to two parallels, if angle b lacks 95° of being 4 times angle a? How many times is the measure contained in angle c? In angle d?



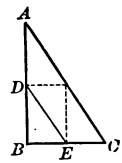
50. Define each term in the following classification of triangles: Equilateral. Isosceles. Scalene.

51. Classify triangles with regard to their angles.

52. The shortest side of a scalene triangle is 7 inches less than the one of medium length, and the one of medium length is 7 inches shorter than the longest. The perimeter is 93 inches. Find each side.

53. Draw a right triangle with a base of 6 centimeters and an altitude of 4 centimeters, and find its area in square millimeters.

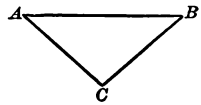
54. In the right triangle ABC, $AB = 12$ inches, $BC = 8$ inches, $BD = 6$ inches, $BE = 4$ inches. The triangle DBE is what fractional part of the triangle ABC?



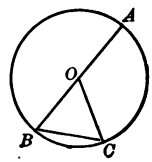
SUGGESTION. — Compare the base and altitude of the small triangles.

55. The vertical angle of an isosceles triangle is 30° larger than either of the base angles. Find each angle.

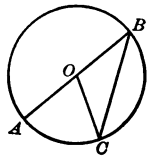
56. ABC is an isosceles triangle; AB is the base. The angle ACB is 15° more than twice ABC. Find all the angles.



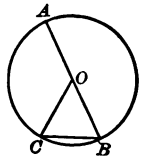
57. In the circle, whose center is O, angle AOC is 112°. How many degrees is angle BOC? Angle OCB? Angle OBC?



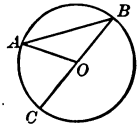
58. In the circle, whose center is O, angle AOC is 70°. How many degrees is angle BOC? Angle OCB? Angle OBC?



59. In the circle, whose center is O, angle AOC is 16° less than twice angle COB. How many degrees is angle OCB?



60. In the circle, whose center is O, the angle AOB is 110°. How many degrees has each angle of the triangle? How many has angle BOC?



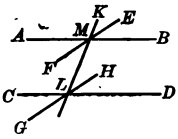
61. Each of the equal sides of an isosceles triangle is 13 inches, and its perimeter is 36 inches. The perpendicular from the vertex to the base is 12 inches. Find the area and perimeter of the triangle formed on each side of the perpendicular.

62. The quadrilateral $ABDC$ has the sides AB and CD parallel. To what special name is it entitled on that account? If AC is 4 inches, BD 1 inch longer than AC, AB twice AC, and CD twice BD, how long is the perimeter of the trapezoid?

63. How many degrees are there in each of five equal angles formed about a given point?

64. How many degrees are there in the angle formed by the bisector of an angle of 50° and the bisector of its supplement?

65. *AB* is parallel to *CD*.
Angle *KMB* = 68°.
FE is a bisector of angle *KMB*.
GH is parallel to *FE*. Find angle *HLD*.



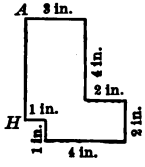
66. Include an acute angle by two lines, one 5 inches, the other 8 inches. Join their extremities. With the joining line as a base draw arcs, and construct a triangle having an 8-inch side adjacent to the 5-inch side of the first triangle. Erase the joining line, and name the quadrilateral thus formed.

67. Draw a rhomboid whose longer sides are each 3 times a shorter side and whose perimeter is 8 inches, and write upon each side its length.

68. Draw a rectangle which is the product of a line 12 centimeters by a line 3 centimeters. Also one which is the square of a line 6 inches. Which has the greater area? The greater perimeter?

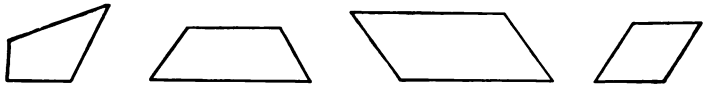
69. Which will require more fencing, a rectangular lot 8 rods by 2 rods, or a square lot containing the same area?

70. The angles being all right angles, how long is the line *AH*? Find the area and perimeter of the surface inclosed.



CHAPTER VIII.

QUADRILATERALS.



TRAPEZIUM. TRAPEZOID. RHOMBOID. RHOMBUS.

1. Draw a quadrilateral having no two sides parallel and write its name upon it.

NOTE. — A quadrilateral having no two sides parallel is called a **Trapezium.**

2. Find the perimeter of the trapezium $ABCD$, AB being twice BC, which is 10 inches, AD being 15 inches longer than BC, CD being 4 times BC.

3. Find the perimeter of the trapezium $ABCD$, whose side AB is 8 inches, AD 4 inches longer than AB, DC 2 inches longer than AD, BC twice AB.

4. In the trapezium $ABCD$, whose perimeter is 50 inches, the side AB is twice BC, AD is 3 times BC, and CD is 4 times BC. How long is each side?

QUERY. — $x = ?$

5. In the trapezium $ABCD$,

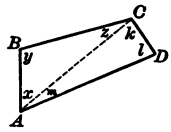
$AD = 4$ times AB.

$CD = 3$ times AB.

$BC = 9$ inches more than AB.

The perimeter of the trapezium is 63 inches. Find each side.

6. The trapezium $ABCD$ is divided by the diagonal AC. What is the sum of the angles of the triangle ABC? Of the triangle ADC? How many degrees are there in all the angles of the quadrilateral? Can you draw a quadrilateral which cannot be divided into two triangles by a diagonal?



PRINCIPLE 28. — *The sum of all the angles of a quadrilateral is equal to four right angles or to 360°.*

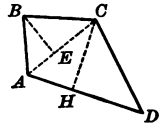
7. In the trapezium $ABCD$ angle A is 60°, angle B is $\frac{4}{3}$ of angle A, angle C is 20° more than angle B. Find angle D.

8. In the trapezium $ABCD$ angle $A = 2$ times angle B; angle $C =$ angle $B + 40°$; angle $D =$ angle $B + 20°$. Find each angle.

SUGGESTION. — Let $x =$ angle B.

9. One of the exterior angles of a trapezium is 79°, another exterior angle is 93°, another 84°. Find all the angles of the trapezium.

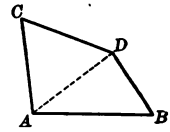
10. Find the area of the trapezium $ABCD$, if the diagonal AC is 7 inches, the altitude BE 4 inches, the altitude CH 6 inches, and the side AD 10 inches.



11. $AB = 12$ centimeters.

The diagonal $AD = 1$ decimeter.

The perpendicular from D to AB is 6 centimeters. The perpendicular from C to $AD = 8$ centimeters. Find the area of the trapezium.

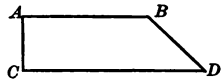


12. Draw a trapezium, divide it into triangles, measure their bases and altitudes, and find the area of the trapezium.

13. Draw a trapezium and a trapezoid and tell what distinguishes one from the other.

14. Draw a diagonal to a trapezoid and show how many degrees there are in the sum of all its angles.

15. AB and CD are each perpendicular to AC. Angle B is 3 times angle D. How many degrees are there in each angle of the trapezoid?



16. Find the perimeter of a trapezoid one of whose parallel bases is $1\frac{1}{3}$ times the length of the other, which is 6 inches, the sum of the non-parallel sides being 21 inches.

NOTE. — Parallel sides of a trapezoid are called **Bases.**

17. In the trapezoid $ABCD$ the sum of the bases is 18 inches and the greater is twice the less; the sum of the non-parallel sides is 9 inches and the greater is twice the less. Find the perimeter and each side of the trapezoid.

18. Draw an isosceles triangle whose base angles are each 80°, and divide it by a line drawn parallel to the base. What figures are formed? How many degrees are there in each angle of the upper figure? In each angle of the lower figure?

NOTE. — When a trapezoid has its non-parallel sides equal it is called an **Isosceles Trapezoid.** The non-parallel sides are called **Legs.**

19. One base of an isosceles trapezoid is 11 centimeters, the other is $7\frac{1}{2}$ centimeters more than one half as

long, and each leg is 7 centimeters shorter than the greater base. Find the perimeter.

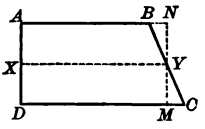
20. Draw an isosceles trapezoid and prolong the non-parallel sides until they meet. What figure have you formed?

21. If your sister is 10 years old and your brother is 14 years old, what is their average age?

22. If you are marked 90% on one examination paper and 100% on another, what is your average %?

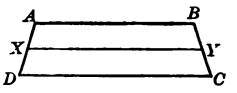
23. If a board is 20 inches wide at one end and 10 inches at the other, what is its average width? Illustrate.

24. Draw the trapezoid *ABCD*, making *AB* 8 inches, *DC* 10 inches, distance between bases 5 inches. What is the length of *XY* drawn midway between the bases? Through *Y* draw *NM* perpendicular to the bases. Cut off the triangle *YMC* and place it in the position of *YNB*. What is the area of the rectangle thus formed? How does it compare with the trapezoid?



NOTE. — A line drawn midway between the bases of a trapezoid is called a **Median.**

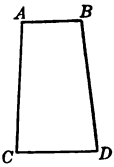
25. *AB*, 10 inches, and *CD*, 12 inches, are bases of a trapezoid. Find the length of the median, *XY*.



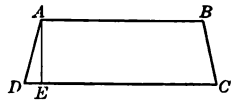
26. The distance between the bases of a trapezoid is 14 centimeters, and the median is 14 centimeters. How many square centimeters are there in its surface?

NOTE. — The perpendicular distance between the bases of a trapezoid is called its **Altitude.**

27. $AB = 6$ inches; $CD = 8$ inches; altitude = 13 inches. What is the area of the trapezoid? What is the perimeter of the rectangle having the same altitude and area?



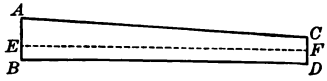
28. $AB = 10$ inches; $DC = 12$ inches; AE, the altitude, $= 4$ inches. Find the area of the trapezoid. Reproduce the trapezoid, cut and replace the parts necessary to transform it into a rectangle. How do you find the average length of the bases of a trapezoid?



PRINCIPLE 29. — *The area of a trapezoid is equal to the product of its altitude and one half the sum of its parallel sides.*

29. How many square feet has a plank 20 feet long, 1 foot wide at one end and 1 foot 6 inches at the other?

30. $AB = 18$ centimeters; $CD = 12$ centimeters; EF, the altitude, $= 125$ centimeters. Find the area of the trapezoid.



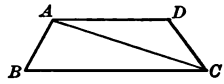
31. The sum of the bases of a trapezoid is 27 inches; the altitude is 5 inches. Find the area.

32. The lower base of a trapezoid is 5 inches more than 3 times the upper base, and the sum of the bases is 37 inches. Find each base. The altitude is $\frac{3}{4}$ of the upper base. Find the area.

33. The lower base of a trapezoid is 6 inches less than twice the upper base, and their sum is 15 inches. The altitude is $\frac{1}{2}$ the lower base. Find the area. If one of

the non-parallel sides is 5 inches and the other is $\frac{6}{7}$ of the upper base, what is the perimeter of the trapezoid?

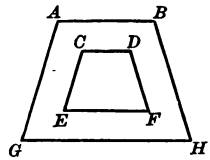
34. Into what triangles does the diagonal AC divide the trapezoid $ABCD$? Draw a construction line showing the altitude of the triangle ABC when BC is the base. Show the altitude of the triangle ADC when AD is the base Which altitude is the greater? Give reasons.



35. AD is 10 inches, BC 14 inches, and the perpendicular distance between the bases 5 inches. Find the area of each triangle and add them to find the area of the trapezoid. Which plan of finding the area of a trapezoid do you prefer, and why?

36. The sum of the bases of a trapezoid is 15 inches, the upper base being twice the lower; the altitude is 4 inches. Find the area of each triangle into which a diagonal divides the trapezoid.

37. $AB = 8$ inches; $GH = 14$ inches; $CD = 4$ inches; $EF = 7$ inches. The altitude of the greater trapezoid is 10 inches, of the less, 5 inches. Find the number of square inches in the space included between their perimeters.



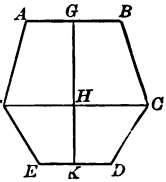
38. The bases of a trapezoid are 9 inches and 13 inches. The area is 55 square inches. How far apart are the bases? What is their average length? How does the difference between the shorter base and their average length compare with the difference between their average length and the longer base?

39. The area of a trapezoid = 45 square inches ; greater base = 12 inches; altitude = 5 inches. Find the smaller base.

40. The area of a trapezoid is 52 square inches; the average length of the bases is 13 inches, and the greater base is 16 inches. Find the other base and the altitude.

41. How much surface is covered by a bay window in the shape of a trapezoid, the longer of the parallel sides being 12 feet, the shorter 6 feet, and the distance between them 3 feet?

42. The irregular hexagon *ABCDEF* has the sides *AB* 8 inches and *ED* 6 inches, each parallel to the diagonal *FC*, 12 inches. *GH*, perpendicular to *FC*, is 7 inches; *HK*, the prolongation of *GH*, is 5 inches. Find the area of the hexagon.



43. Draw two equal parallel lines and join each extremity of one to that extremity of the other which is the nearer. The surface thus inclosed is bounded by how many lines? Are both pairs of opposite sides parallel? To what name is the figure entitled by the parallelism of its opposite sides?

SUGGESTION. — See Chap. V., p. 52.

44. Find the perimeter of a parallelogram, in which one side is 6 inches, and the sides adjacent to it are each $2\frac{1}{2}$ times as long.

45. Draw a rectangular parallelogram whose adjacent sides are 7 centimeters and 5 centimeters, and find its perimeter and area.

46. Draw a parallelogram, not rectangular, the longer sides of which are each twice a shorter side, the shorter

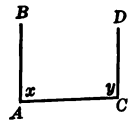
sides being each 2 centimeters. How long is the perimeter of the parallelogram? What name is given to a parallelogram whose angles are oblique?

47. Find the perimeter of a rhomboid each of whose long sides is equal to the sum of the shorter sides, a short side being 20 millimeters.

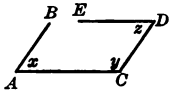
48. The perimeter of a rhomboid is 56 feet. Each long side is 3 times a short side. Find each side.

49. Is a rectangle a parallelogram? Is a square? Give reasons.

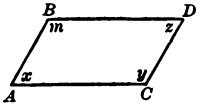
50. *AB* and *CD* are parallel. What name belongs to *AC*? If angle $x = 87°$, what is angle y? Quote a geometric principle which helps you to your answer.



51. *AB* and *CD* are parallel. *ED* is parallel to *AC*. If $x = 50°$, how many degrees is y? How many is z? Compare angle x and angle z.



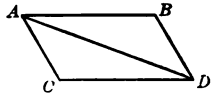
52. In the parallelogram *ABCD*, if $x = 60°$, how many degrees is the angle y? The angle z?



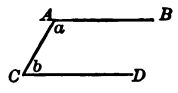
53. Draw a parallelogram and show the truth of the following principle:

PRINCIPLE 30. — *The opposite angles of a parallelogram are equal.*

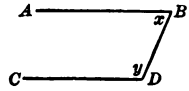
54. The opposite sides of a parallelogram are equal. Are the triangles *ACD* and *ABD* equal in all their parts? Show the truth of your answer by referring to a geometric principle.



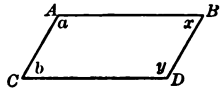
55. *AB* and *CD* being parallel, what is the sum of the angles *a* and *b*? Quote the geometric principle.



56. *AB* and *CD* being parallel, how many degrees are there in angle x + angle y?



57. How many degrees are there in all the angles of the parallelogram *ABCD*? Give reasons.



58. One of the angles of a rhomboid is 73°. Find the number of degrees in each of the remaining angles.

59. One of the angles of a rhomboid is twice an adjacent angle. How many degrees are there in each angle of the rhomboid?

60. One of the angles of a parallelogram is 3 times an adjacent angle. How many degrees are there in each angle of the parallelogram?

61. Can you show that the sum of all the angles of a parallelogram equals 360°, using the figure in Ex. 57?

62. One of the angles of a parallelogram is equal to its adjacent angle. How many degrees has each angle of the parallelogram? What kind of a parallelogram is it?

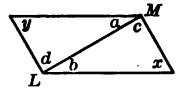
63. Draw a rhomboid, having its longer sides each 10 inches and its shorter sides each 25% of the length of a longer side, and find its perimeter.

64. Make an angle of 80° by two lines, one 5 inches and the other 4 inches. Draw parallels to those lines, forming a rhomboid. How many degrees has each of the other angles of the rhomboid?

65. The sum of the obtuse angles of a rhomboid is twice the sum of the acute angles. Find each angle.

66. How does a rhomboid differ from a rectangle?

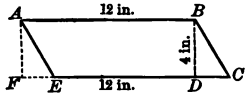
67. If angle a, formed by the diagonal and one side of the rhomboid, is 30°, how many degrees is angle b? If angle c is 100°, how many is angle d? Give authority. If angle x is 50°, how many is angle y? Quote geometric principle.



68. The longer sides of a rhomboid are each 8 centimeters longer than a short side, and the perimeter is 36 centimeters. Find the sides.

69. The longer sides of a rhomboid are each 3 inches more than 5 times as long as a shorter side, and the perimeter is 54 inches. Find all the sides.

70. Reproduce $ABCE$, draw the perpendicular BD, cut off the triangle BDC, and place it in the position of AEF. What is the area of the rectangle $ABDF$? Compare it with the parallelogram $ABCE$. Compare the base and altitude of the rectangle with the base and altitude of the parallelogram.

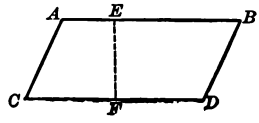


NOTE. — The distance between parallel sides of a parallelogram is called the **Altitude**.

71. Can a parallelogram have more than one altitude? Illustrate.

PRINCIPLE 31.— *The area of a parallelogram is equal to that of a rectangle having the same base and altitude.*

72. $AB = 11$ inches. EF, which measures the distance between AB and CD, is 5 inches. Find the area of the rhomboid.



73. Find the area of a rhomboid whose perimeter is 36 millimeters, whose shorter sides are each 6 millimeters, and whose altitude perpendicular to a long side is 4 millimeters.

74. The perimeter of a rhomboid = 42 inches; a long side is twice an adjacent side; the altitude perpendicular to a long side = 6 inches. Find the area of the rhomboid.

75. The area of a parallelogram = 54 centimeters; the base = 9 centimeters. Find the altitude.

76. A base of a rhomboid is 4 centimeters longer than the altitude perpendicular to it, and their sum is 16 centimeters. Find the area of the rhomboid.

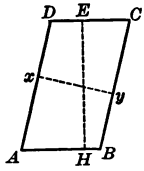
77. A base of a rhomboid plus the altitude perpendicular to it equals 32 inches. The base is 8 inches longer than the altitude. Find the area of the rhomboid.

SUGGESTION. — Let x = altitude; ? = base.

78. A base of a rhomboid is 5 inches longer than the altitude perpendicular to it. The same base plus the same altitude is 19 inches. Find its area.

79. The long diagonal of a rhomboid is 12 inches. One of the short sides is one half the diagonal, and the perimeter of the triangle formed by the diagonal and two adjacent sides is 29 inches. Find the perimeter of the rhomboid.

80. $AB = 5$ inches, and EH, perpendicular to it, $= 8$ inches. What is the area of the rhomboid $ABCD$? What is the product of BC and its perpendicular xy?



81. The adjacent sides of a rhomboid are 12 inches and 10 inches respectively, and the altitude

perpendicular to the longer side is 5 inches. Find the length of the altitude perpendicular to the shorter side.

82. Place two equilateral triangles so that a side of one coincides with a side of the other. The parallelogram thus formed is a rhombus. If a side of the equilateral triangle is 8 inches, what is the perimeter of the rhombus?

NOTE. — A rhomboid whose sides are all equal is called a **Rhombus.**

83. Draw two triangles each having an angle of 40° included by sides each 5 inches. Place the sides opposite the given angles together, and find the perimeter of the rhombus thus formed.

84. Have you ever seen panes of glass in the shape of a rhombus? If so, where? What other name is given to that shape?

85. Is the "diamond" of the baseball field a rhombus or a square?

86. How does a rhombus differ from a square? From other rhomboids?

87. If you have a square frame 4 inches each way, and press together two opposite corners, what shape will the frame take? Will its inclosed space grow less or remain the same? Which is the greater, a square whose sides are each 4 inches, or a rhombus whose sides are each 4 inches?

88. To what six names is a rhombus entitled?

89. Draw a rhombus whose angles are equal to those given in Ex. 82. What kind of triangles are formed by its short diagonal? How many degrees are there in each angle of the triangles thus formed? How many in each of the rhombus?

90. What is one side of a rhombus whose perimeter is 40 centimeters?

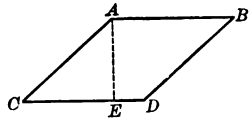
91. Draw and cut out three rhombuses whose angles are equal to those given in Ex. 82. Place them about a common point, the vertex of a large angle of each coinciding with the vertex of a large angle of the others. What figure is thus formed?

92. What kind of a triangle is formed by a diagonal and two sides of a rhombus?

93. What is the sum of all the angles of a rhombus? Quote authority.

94. How does the number of degrees in all the angles of a rhombus compare with the number in all the angles of a triangle?

95. One side of the rhombus *ABCD* is 12 centimeters and the altitude *AE* is 8 centimeters. Find the area.



96. What is the area of a rhombus whose perimeter is 32 centimeters and the altitude $\frac{3}{4}$ of one side?

97. Find the area of a rhombus in which the sum of the altitude and one side is 19 inches and the perimeter is 40 inches.

98. Find the area and perimeter of a rhombus in which one side is 3 inches more than the altitude, and their sum is 15 inches.

99. The area of a rhombus is 90 square inches and one of its sides is 10 inches. How far apart are the parallel sides?

100. The area of a rhombus is 35 square centimeters; the altitude is 5 centimeters. Find the perimeter.

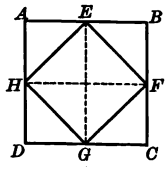
101. The area of a rhombus is 120 square inches and its perimeter is 48 inches. How far apart are its opposite sides?

102. Find one side of a rhombus in which the perimeter of the triangle formed by two sides and the long diagonal is 22 inches, the long diagonal being 10 inches.

103. Find the perimeter of a rhombus in which the short diagonal is 10 inches and the perimeter of one of the triangles into which it divides the rhombus is 36 inches.

104. One angle of a rhombus is 60°. Find all the others. Reproduce the rhombus and draw its short diagonal. Into what kind of triangles is the rhombus divided by its short diagonal?

105. The square $ABCD$ is 8 inches in dimensions. E, F, G, and H are the middle points of their respective lines. What is the area of the square $EFGH$?

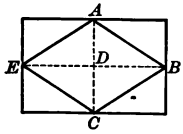


SUGGESTION. — Draw construction lines EG and HF and compare the triangles thus formed.

106. How long is each diagonal of the inscribed square? How does the product of the diagonals of the inscribed square compare with the area of the square in which it is inscribed?

107. Show that the area of a square is equal to one half the square of one of its diagonals.

108. Show that the area of a rhombus is equal to one half the product of its diagonals.



109. Draw a rectangle 6 inches by 4. Connect the middle points of the adjacent sides. How long is the long diagonal of the rhombus thus formed? Its short diagonal? What is the area of the rectangle? The area of the rhombus is what fractional part of the area of the rectangle?

110. The short diagonal of a rhombus is 10 inches and the long diagonal is $1\frac{1}{3}$ times as long. Find the area.

111. One of the diagonals of a rhombus is 7 inches longer than the other, and their sum is 27 inches. Find the area.

112. The area of a rhombus is 144 square inches. One of its diagonals is 16 inches. Find the other.

QUERY. — What is the area of the rectangle in which the rhombus is inscribed?

113. The perimeter of the triangle formed by the long diagonal and two sides of a rhombus is 36 inches. The perimeter of the triangle formed by the short diagonal and two sides is 32 inches. The perimeter of the rhombus is 40 inches. Find its area.

114. The diagonals of a rhombus are 6 inches and 8 inches and one of its sides is 5 inches. What is the distance from that side to the opposite side?

QUERY. — Given the area and base of a parallelogram, how do you find the altitude?

115. Is every rhombus a rhomboid? Is every rhomboid a rhombus?

116. Can you inscribe a rhombus in a circle?

117. Draw a square and a rhombus whose sides are each 3 inches long and tell which is the larger.

SUGGESTION. — Make the rhombus in such a way that the long diagonal will be very long and the short diagonal very short.

118. Draw a rhombus whose base and altitude are equal, if you can. Explain.

119. Define each term in the following classification:

Quadrilateral { Trapezium. / Trapezoid. / Parallelogram { Rectangle — Square. / Rhomboid — Rhombus :

CHAPTER IX.

RATIO AND PROPORTION.

1. 6 is how many times 3? 15 is how many times 5?

2. What quotient is obtained by dividing 28 by 7? By dividing 3 by 5?

NOTE. — The quotient obtained by dividing one quantity by another of the same kind is called their **Ratio.**

3. What is the ratio of 10 to 5? Of 21 to 7? Of 8 to 4? Of 8 to 8? Of 8 to 16?

4. Complete the following statements:

The ratio of 10 to 2 is —. The ratio of 10 to 8 is —.
The ratio of 10 to 15 is —. The ratio of 12 to 8 is —.

NOTE. — Instead of writing out the whole statement, we use signs to express ratio. $10 : 5 = 2$ is read — The ratio of 10 to 5 is 2.

Find the ratios of:

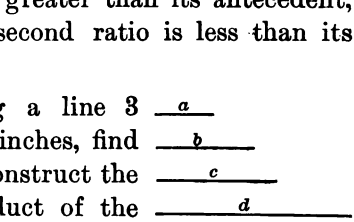
5. 27 : 9
6. 16 : 6
7. 21 : 7
8. 5 : 10
9. 8 : 12
10. 9 : 6
11. 10 : 15
12. 21 : 28
13. 36 : 24
14. 25 : 15

15. $\frac{m}{n}$ If the line m is 14 feet long and the line n is 21 feet long, what is the ratio of m to n?

53. Write a true proportion in which the first antecedent is greater than its consequent.

54. Can you write a true proportion in which the consequent of the first ratio is greater than its antecedent, and the consequent of the second ratio is less than its antecedent? Give reasons.

55. $a : b = c : d$. a being a line 3 inches, b, 5 inches, and c, 6 inches, find the length of the line d. Construct the rectangle which is the product of the lines a and d, and also that which is the product of the lines b and c. Compare their areas.



56. $A : B = C : D$. If A is a line 8 inches, B, 6 inches, and C, 4 inches, how long is D? Find the difference of the areas of rectangle AD and rectangle BC. Find the difference of the perimeters of those rectangles.

NOTE. — The use of ratio and proportion is not confined to geometry. Comparing the value of 1 hat at \$2.00 with the value of 5 hats at the same price, we see that the more hats we buy at a certain price the more money we must pay. 1 hat : 5 hats = \$2.00 : \$10.00.

57. Complete the proportion which expresses the relation of 5 hats, 7 hats, and their values, when 5 hats are worth \$15.00. 5 hats : 7 hats = \$15.00 : ?

58. Complete the proportion, 2 yards of silk : 7 yards of silk = \$6.00, the price of 2 yards : ?

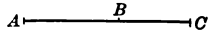
59. If 1 apple costs 2 cents, 1 apple : 7 apples = 2 cents : ? 3 apples : 8 apples = 6 cents : ?

60. What is the ratio of an arc of 120° to the semicircumference of which it is a part?

61. The line a is 8 centimeters. The sum of the lines a and b is 20 centimeters. What is the ratio of the less

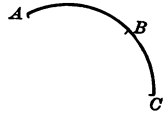
line to the greater? How many of the shorter lines equal three of the longer lines?

62. If BC is 6 inches, and $AB : BC = 4 : 3$, how long is AB? Find the ratio of AC to BC. Of AB to AC.

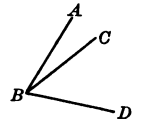


63. If AC is 15 inches, and $AC : BC = 3 : 1$, what is the ratio of AB to BC?

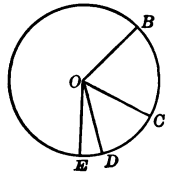
64. If the arc AC is 30 inches, and the arc AB is 20 inches, what is the ratio of arc AC to arc BC? Of arc AB to arc BC?



65. If angle ABC is 20°, and angle CBD is 50°, what is the ratio of the greater to the less? Of the less to the greater? Of the less to their sum? Of the greater to their difference? Find the ratio of an angle of 50° to its complement. To its supplement. Find the ratio of the complement of an angle of 50° to its supplement.



66. In the circle whose center is O, angle BOC : angle $COD = 3 : 2$. The arc BC is 21 inches. Find the arc CD. Arc CD : arc $DE = 3 : 1$. How long is the arc DE? If the arc BC were 15 inches, how long would be the arc DE?



67. $a : b = c : d$. $a =$ an angle of 40°, b an angle of 70°, c an angle of 80°. Find the number of degrees in the angle d.

68. Given the ratio 7 : 5. Multiply both terms by 10, giving the ratio 70 : 50. Compare it with the original ratio. Is it true that $70 : 50 = 7 : 5$? Can you obtain a true proportion in the same way by using as a multiple any other number than 10?

69. Experiment by multiplying both terms of ratios by the same number until you are convinced of the truth of the following statement:

PRINCIPLE 33. — *Two quantities are in the same ratio as their equimultiples.*

QUERY. — What are equimultiples?

70. The areas of two triangles are to each other as 4 to 5. What is the ratio of the areas of the rectangles whose bases and altitudes are respectively equal to those of the triangles? If the area of the smaller triangle is 20 square inches, what is the area of each of the other three figures?

71. What is the ratio of the circumferences of two circles, the diameter of one being 7 inches, and of the other 14 inches?

72. Find the ratio of the circumferences of two circles, the radius of one being 20 inches, and of the other 10 inches.

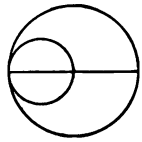
73. With radii of different lengths construct several circles. Compare their circumferences, and illustrate the following principle:

PRINCIPLE 34. — *The circumferences of circles are in the same ratio to each other as their diameters or their radii.*

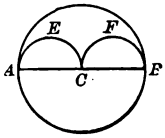
74. Are the diameters of circles equimultiples of their radii? Are the circumferences equimultiples of diameters? Of radii? Show the truth of your answers.

75. Two circumferences are in the ratio of 3 to 1. The diameter of the smaller circle is 17 inches. Find that of the greater.

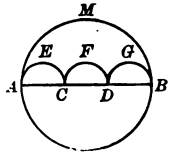
76. The radius of the greater circle is the diameter of the smaller. What is the ratio of their circumferences? If the circumference of the greater is 6 centimeters, what is the circumference of the smaller?



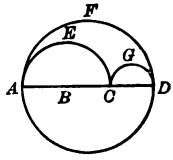
77. C is the center of the circle; AEC and CFB are semicircumferences. If the circumference of the circle whose center is C is 30 millimeters, what is the sum of the arcs AEC and CFB?



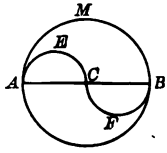
78. Three equal semicircles are placed upon a line AB, and the semicircumference AMB is drawn. If the arc AMB is 36 inches, what is the sum of the arcs AEC, CFD, and DGB?



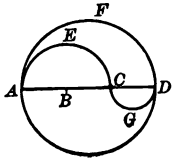
79. $AB = BC = CD$. If the semicircumference AEC is 12 centimeters, how long is the semicircumference CGD? How long is the semicircumference AFD?



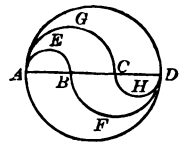
80. C is the center of the circle. If its circumference is 40 centimeters, how long is the double curve $AECFB$, composed of two semicircumferences? The irregular figure $AECFBM$ is what part of the circle?



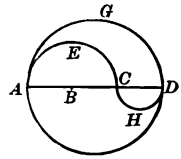
81. $AB = BC = CD$. If the semicircumference AFD is 15 centimeters, how long is the semicircumference CGD? How long is the double curve composed of the two circumferences AEC and CGD?



82. The diameter AD is divided equally at the points B and C, and the double curves are each composed of semicircumferences. If the circumference of the circle is 30 centimeters, how long is the perimeter of the irregular figure $AGCHDFBE$?



83. The diameter AD is trisected at the points B and C. $AD = 21$ inches. AEC and CHD are semicircumferences. Find the length of the perimeter of the irregular figure $AECHDG$.



84. The side of one square is 3 feet, of another 6 feet. What is the ratio of the perimeters of those squares?

85. The side of one square is 2 feet, of another 1 yard. Find the ratio of their perimeters.

86. Find the ratio of the perimeters of two equilateral triangles, a side of one being 8 inches and of the other 10 inches.

87. Find the ratio of the perimeters of two regular hexagons, a side of one being 4 inches and of the other 7 inches.

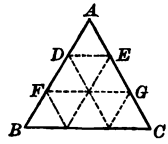
88. Draw two squares, a side of one being twice as long as a side of the other, and find the ratio of their areas.

89. Find the ratio of the areas of two squares, a side of one being 3 times as long as a side of the other.

90. Find the ratio of the areas of two squares whose sides are in the ratio of 4 : 1.

91. What is the perimeter of a square each side of which is 3 times as long as the side of another square whose perimeter is 20 inches?

92. The equilateral triangle ABC has each side 3 times as long as the homologous side of the equilateral triangle ADE. What is the ratio of their areas?



SUGGESTION. — Reproduce and fold the larger triangle according to the indicated lines.

93. Build from inch cubes a cube 2 inches each way. How many inch cubes make the cube? Build a 3-inch cube. How many layers of inch cubes are there? How many inch cubes are in each layer? Find the ratio of the volumes or contents of a 2-inch cube and a 3-inch cube.

94. Find the ratio of the volumes of a 3-inch cube and a 4-inch cube. Find the ratio of their surfaces.

95. Find the ratio of a cube whose edge is 2 inches to one whose edge is 4 inches.

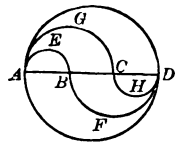
96. Find the ratio of a cube, one of whose faces contains 9 square inches, to a cube, each of whose faces contains 16 square inches.

97. Find the ratio of a cube, the sum of whose edges is 24 inches, to that of a cube, the sum of whose edges is 60 inches.

QUERY. — How many edges has a cube?

98. Find the ratio of a parallelopiped 8 inches long, 5 inches wide, and 4 inches deep to one 10 inches long, 6 inches wide, and 2 inches deep.

82. The diameter AD is divided equally at the points B and C, and the double curves are each composed of semicircumferences. If the circumference of the circle is 30 centimeters, how long is the perimeter of the irregular figure $AGCHDFBE$?



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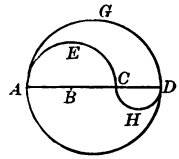


84. The side of one square is 3 feet, of another 6 feet. What is the ratio of the perimeters of those squares?

85. The side of one square is 2 feet, of another 1 yard. Find the ratio of their perimeters.

86. Find the ratio of the perimeters of two equilateral triangles, a side of one being 8 inches and of the other 10 inches.

87. Find the ratio of the perimeters of two regular hexagons, a side of one being 4 inches and of the other 7 inches.

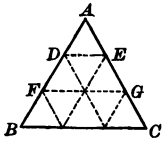
88. Draw two squares, a side of one being twice as long as a side of the other, and find the ratio of their areas.

89. Find the ratio of the areas of two squares, a side of one being 3 times as long as a side of the other.

90. Find the ratio of the areas of two squares whose sides are in the ratio of 4 : 1.

91. What is the perimeter of a square each side of which is 3 times as long as the side of another square whose perimeter is 20 inches?

92. The equilateral triangle ABC has each side 3 times as long as the homologous side of the equilateral triangle ADE. What is the ratio of their areas?



SUGGESTION. — Reproduce and fold the larger triangle according to the indicated lines.

93. Build from inch cubes a cube 2 inches each way. How many inch cubes make the cube? Build a 3-inch cube. How many layers of inch cubes are there? How many inch cubes are in each layer? Find the ratio of the volumes or contents of a 2-inch cube and a 3-inch cube.

94. Find the ratio of the volumes of a 3-inch cube and a 4-inch cube. Find the ratio of their surfaces.

95. Find the ratio of a cube whose edge is 2 inches to one whose edge is 4 inches.

96. Find the ratio of a cube, one of whose faces contains 9 square inches, to a cube, each of whose faces contains 16 square inches.

97. Find the ratio of a cube, the sum of whose edges is 24 inches, to that of a cube, the sum of whose edges is 60 inches.

QUERY. — How many edges has a cube?

98. Find the ratio of a parallelopiped 8 inches long, 5 inches wide, and 4 inches deep to one 10 inches long, 6 inches wide, and 2 inches deep.

99. A box 9 inches long, 7 inches wide, and 3 inches deep will contain how many times as much as a box 3 inches long, 1 inch wide, and 3 inches deep?

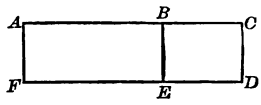
100. Find the ratio of the areas of two rectangles, one being 8 centimeters long and 5 centimeters wide, the other being 10 centimeters long and 6 centimeters wide.

101. Draw rectangles of different dimensions and illustrate by numbers the truth of the following principle:

PRINCIPLE 35. — *The areas of any two rectangles are in the same ratio as the products of their bases by their altitudes.*

102. Solve the proportion $8 \times 4 : 6 \times 4 = 8 : ?$

103. $AB = 7$ inches; $BC = 4$ inches; $AF = 3$ inches. Find the ratio of the rectangles $ABEF$ and $BCDE$. Of $ACDF$ and $BCDE$.



104. Draw rectangles with equal altitudes and bases of different lengths and illustrate the following principle:

PRINCIPLE 36. — *Two rectangles having equal altitudes are proportional to their bases.*

105. Draw rectangles having equal bases and different altitudes and illustrate the following principle:

PRINCIPLE 37. — *Two rectangles having equal bases are proportional to their altitudes.*

106. What is the ratio of the area of a triangle whose base is 10 centimeters and altitude 8 centimeters to that of a triangle whose base is 12 centimeters and altitude 5 centimeters?

107. Triangle ABC has a base 10 inches, altitude 7 inches. Triangle DEF has a base 5 inches, altitude 10 inches. Find the ratio of their areas.

108. Compare areas of triangles until you are able to complete the statement:

The areas of two triangles are to each other as the products of _____?

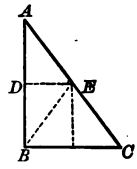
109. Find the areas of triangles having equal bases and differing altitudes until you can complete the statement:

Two triangles with equal bases are to each other as _____?

110. Find the areas of triangles having equal altitudes and differing bases until you can complete the statement:

Two triangles having equal altitudes are to each other as _____?

111. Reproduce the triangle ABC, right-angled at B. If AB is 8 inches and BC is 6 inches, AC will be 10 inches (a fact which you will soon learn to prove). At D, the middle point of AB, draw DE parallel to the base. Fold the triangle along the dotted lines, and show that the four triangles thus formed are equal. What is the ratio of AD to AB? Of AE to AC? Of DE to BC?



112. Prove by reference to a geometric principle that angle ADE = angle ABC. That angle AED = angle ACB.

113. Draw a triangle one of whose sides (not a base) is 12 centimeters. At different points in the side draw lines parallel to the base, measure, and illustrate the following statement:

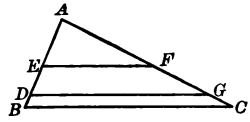
PRINCIPLE 38. — *A line drawn parallel to the base of a triangle divides the sides proportionally.*

114. EF and DG are parallel to BC.

$AB = 6$ inches. $AC = 12$ inches.

$AE = 3$ inches. $ED = 2$ inches.

Find AF, FG, and GC.



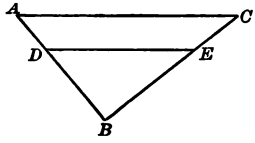
115. $AB = 15$ centimeters.

$BC = 18$ centimeters.

DE is parallel to AC.

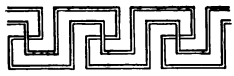
$AD = 5$ centimeters.

Find EC and EB.



CHAPTER X.

CUMULATIVE REVIEW No. 3.



1. The above is one pattern of an ornamental border called the "Greek Fret." How many right angles are there in the piece here represented?

2. Draw a rectangle which is the product of a line 9 inches and a line 5 inches, and a rhomboid which has the same area. Which has the greater perimeter?

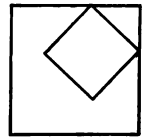
3. Lay off a line 5 inches on a line 8 inches, and construct the square of their difference.

4. Cut out the square of a line 6 inches from one corner of the square of a line 8 inches, and find the area and perimeter of the irregular hexagon which is their difference.

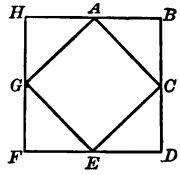
5. How many times can the G. C. M. of a line 35 inches and one 20 inches be laid off on the shorter line?

6. How many times can the longer of two lines, one 8 inches and the other 12 inches, be laid off on their L. C. M.?

7. Is the smaller square inscribed in the greater? Give reasons for your answer.



8. A, C, E, and G are the middle points of their respective lines. Reproduce and show what part the inscribed square is of the outer square.



9. If the side HB is 8 inches, what is the area of the inscribed square? If AB equals 5 inches, what is the area of the inscribed square?

10. Find the area and perimeter of the rectangle whose longer sides are each the sum of two lines, 11 centimeters and 4 centimeters, and whose shorter sides are each the difference of those lines.

11. Find the area and perimeter of the rectangle which is the product of the sum and the difference of two lines, one 12 inches, the other 8 inches.

12. Three arcs compose a circumference. The first is 5 times the second, and the third is 4 times the second. How many degrees are there in each? How long is each arc if the radius is 2 feet 11 inches?

13. How many millimeters are there in the circumference of a circle whose radius is $3\frac{1}{2}$ centimeters?

14. A degree of a circumference whose radius is 21 inches is what fractional part of a degree of a circumference whose radius is 42 inches?

15. How many degrees are there in an arc which contains 30° more than its supplement?

QUERY. — Will you let x = the arc or the supplement?

16. How long is an arc of 72° of a circumference whose diameter is 35 millimeters?

17. Find the perimeter of a sector which is $\frac{1}{10}$ of a circle whose diameter is $17\frac{1}{2}$ inches.

18. Make ten angles of 36 each around a common point by lines 5 inches long. Join the extremity of each line to the extremities of the neighboring lines. Is the polygon thus formed a regular polygon? Write its name upon it.

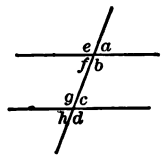
NOTE. — A polygon of ten sides is called a **Decagon.**

19. Divide the circumference of a circle whose diameter is 8.4 inches into two arcs, one of which is 7 times the other.

20. What is the ratio of a circumference to an arc whose remaining arc is 288° greater?

21. Divide a circle into sectors whose arcs are 120° (by making central angles with a protractor), and show the ratio of each sector to the circle.

22. Angle b = angle $a + 40°$. Find all the angles formed by the transversal and the parallels.



23. How many degrees are there in each of the three angles formed at a given point on the same side of a straight line if the first is twice the second and the second is 3 times the third?

SUGGESTION. — $x = ?$

24. The shortest side of a scalene triangle is 15 inches. Another side is $33\frac{1}{3}\%$ longer than the first. The third side is 25% longer than the second. Find the perimeter. Find the ratio of the shortest side to the longest.

25. One side of a scalene triangle whose perimeter is 142 decimeters is 17 decimeters longer than one of its adjacent sides and 21 decimeters longer than the other side. What is its ratio to the perimeter?

26. Each of the equal sides of an isosceles triangle is 18 inches. The base is $33\frac{1}{3}\%$ of one of the equal sides. Find the perimeter. What is the ratio of the perimeter to the base?

27. Find the perimeter of the isosceles triangle whose base is 14 inches, and whose equal sides are each 25% longer than the base.

28. Find the area of an isosceles triangle whose perimeter is 32 inches, each of whose equal sides is $12\frac{1}{2}$ inches, and whose altitude is 12 inches.

29. An exterior vertical angle of an isosceles triangle is 7 times the vertical angle. Find each angle of the triangle and the ratio of the base angle to the vertical angle.

30. Construct an equilateral triangle each of whose sides is 5 inches, and upon each side construct an equilateral triangle and find the perimeter of the figure thus formed. Write the name of the figure. The original triangle is what fractional part of it?

31. Each of the base angles of an isosceles triangle is 3 times the vertical angle. How many degrees are there in each angle of the triangle?

32. In a right-angled scalene triangle the larger acute angle is 10 times the smaller. Find the number of degrees in each angle.

33. How many degrees are there in two consecutive angles of a rhomboid?

34. How many times will a square whose side is 2 inches be contained in a square whose side is 12 inches?

35. An angle of a rhomboid is 5 times an adjacent angle. How many degrees is each angle of the rhomboid?

36. The rhomboid $ABCD$ is 78 inches in perimeter, and the sum of the longer sides is twice the sum of the shorter sides. Find the ratio of two adjacent sides.

37. Find the area of a rhomboid whose perimeter is 40 inches, its shorter sides being each 7 inches, and the perpendicular distance between the longer sides 6 inches.

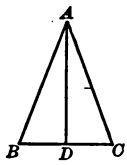
38. When all the sides of a rhomboid are equal, what is the figure called? If the perimeter of a rhombus is 20 inches, what is one side?

39. What is the ratio of the area of a rhombus to that of the rectangle of its diagonals? Illustrate by diagram.

40. Find the area of a rhombus, one of whose diagonals is 6 inches longer than the other, and their sum, 20 inches.

41. Draw two triangles having two sides and the included angle of one respectively equal to two sides and the included angle of the other, and compare their areas.

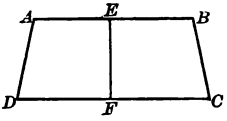
42. $AD = 24$ inches; $BC = 20$ inches; $AB = 26$ inches. AD is perpendicular to BC at its middle point. Find the perimeter and area of the triangles ADC and ABC



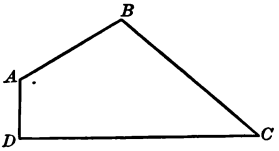
43. Draw two lines, 8 inches and 6 inches, cutting each other perpendicularly at their middle points. Join each extremity of the lines to the extremity of its neighboring line. Can you prove that the figure thus

formed is a rhombus? Quote the geometric principles which apply.

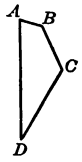
44. $AB = 10$ inches; $DC = 12$ inches; the altitude $= 5$ inches; find the area of the trapezoid.



45. The trapezium $ABCD$ has the side AB 7 inches longer than the side AD, and 7 inches less than the side BC; DC is 7 inches longer than the side BC; the perimeter is 70 inches. What is the ratio of the side AD to the perimeter?



46. The side AD of the trapezium $ABCD$ is 5 centimeters longer than DC; DC is 4 centimeters longer than BC; BC is 3 centimeters longer than AB; the perimeter is 50 centimeters. Find each side.

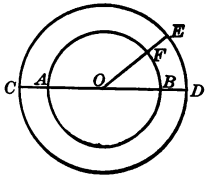


47. Can you draw a trapezium such that one of its diagonals will divide it into two equal triangles?

SUGGESTION. — Kite.

48. When a carriage is moving, which moves in space the more rapidly, a point on the hub of a wheel, or a point on the tire? Draw curves showing the path of each.

49. The diameter $AB = 14$ inches; $CD = 21$ inches; angle $EOD = 40°$; find the length of arc ED and arc FB. Compare the ratio of the arcs with the ratio of the circumferences. With the ratio of the diameters.



50. If the diameter *AB* were 10 inches, *CD* 15 inches, and the arc *FB* were 8 inches, what would be the length of the arc *ED*?

51. If *OB* were 9 inches, *OD* 15 inches, the arc *ED* 12 inches, how long would the arc *FB* be?

52. If the circumference of the larger circle were 50 inches, that of the smaller 30 inches, and the arc *ED* 5 inches, what would be the length of the arc *FB*?

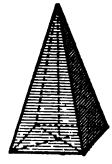
53. Cut out six squares, the dimensions of each being 1 decimeter, and fasten them together (by gluing or sewing) so as to make a cubic decimeter. How many centimeters measure the length of all the edges of the figure?

NOTE. — A cubic decimeter is called a **Liter.**

54. How many square centimeters are there in all the faces of a liter? How many in all the squares that can be inscribed in the faces of a liter?

55. Draw a square decimeter, and at the middle point of each side draw a perpendicular 2 decimeters long. From the extremity of each perpendicular draw oblique lines to the side of the decimeter upon which the perpendicular stands. Find the area of the polygon thus formed.

56. Cut out the figure described in Ex. 55, and fold the four triangles back so that their vertices meet in a common point, and they form a rectangular pyramid. Find the area of the lateral or side surfaces of the pyramid. Of all its surfaces.



57. Place this pyramid upon a liter so that its base coincides with a side of the liter, and find the lateral surfaces of the solid thus formed.

CHAPTER XI.

POLYGONS.

NOTE. — A **Polygon** is a plane figure bounded by straight lines. A **Regular Polygon** is one which has all its sides and all its angles equal. A polygon of three sides is called a **Triangle**; of four sides a **Quadrilateral**; of five sides a **Pentagon**; of six sides a **Hexagon**; of seven sides a **Heptagon**; of eight sides an **Octagon**; of nine sides a **Nonagon**; of ten sides a **Decagon**; of twelve sides a **Dodecagon.**

1. What plane figure have you studied that is not a polygon?

2. Construct a regular polygon of three sides, each of which is 8 centimeters, and write its name upon it.

3. Draw a regular polygon of four sides and write its name upon it.

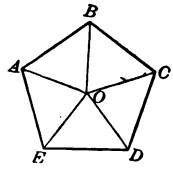
4. Make five equal angles around a common point by equal lines, and join the extremity of each line to the extremities of the adjacent lines. Can you prove that the joining lines are equal?

SUGGESTION. — See Geom. Prin. 24.

The surface inclosed by the joining lines forms what kind of a polygon?

5. Draw a circle whose radius is one of the equal lines which form the angles at the center of the figure which you drew for Ex. 4. Will it touch the vertices of all the angles of the polygon? Argue for your statement.

6. Reproduce the pentagon whose center is O, the angles at O being equal, and formed by equal lines. What kind of a triangle is OBC? OBA? Are they equal triangles? Why? How many degrees has each angle of the triangle AOE? How many degrees has angle EAB? Angle ABC? Each angle of the pentagon? Is it a regular pentagon?



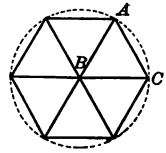
7. Reproduce the figure in Ex. 6, and draw lines from the center to the middle point of each side. Can you prove those lines to be perpendicular to the sides? Are they equal?

NOTE. — A line drawn from the center of a regular polygon to the middle point of one of its sides is called its **Apothem.**

8. With an apothem as a radius, describe a circumference. Will it include the extremities of all lines drawn from the center to the middle point of a side? Prove. Are the sides of the polygon tangent to the circle?

9. Show how you inscribe a circle in a regular polygon.

10. Place six equal equilateral triangles around the common point B. With B as a center, and a radius equal to a side of one of the triangles, circumscribe a circumference about the hexagon thus formed. How may you know that each vertex will be in the circumference?



11. Which is longer, BA or AC? Give reason.

12. How does the side of a regular hexagon compare with the radius of the circle in which the hexagon is inscribed?

PRINCIPLE 39. — *The side of a regular hexagon is equal to the radius of the circle in which it is inscribed.*

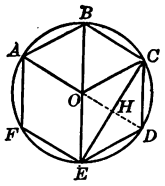
13. How many times does the side of a regular hexagon lie as a chord around the circumscribed circumference?

14. Draw a circle with a radius of 6 inches and inscribe a regular hexagon.

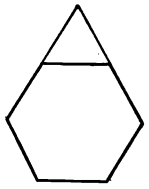
15. Inscribe a regular hexagon in a circle whose radius is 7 inches. Join the alternate vertices with the center, forming three polygons. What kind of polygons are they? How long is the perimeter of each?

16. The sum of the three radii bears what ratio to the perimeter of the hexagon? The perimeter of each rhombus bears what ratio to the perimeter of the hexagon?

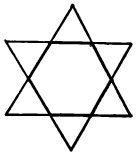
17. *ABCDEF* is a regular hexagon. Which is the greater, the triangle *BOC* or the triangle *CDE*? Give reasons. Can you prove that *OH*, the altitude of the triangle *OCE*, equals *HD*, the altitude of the triangle *CDE*? If the radius of the circle is 12 inches, how long is *OH*? *AH*?



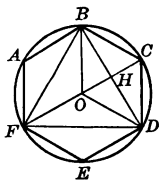
18. Construct an equilateral triangle upon one side of a regular hexagon. The area of the irregular pentagon thus formed bears what ratio to that of the hexagon? What is the ratio of their perimeters?



19. Construct an equilateral triangle upon each side of a regular hexagon. The area of the six-pointed star thus formed bears what ratio to the hexagon? What is the ratio of their perimeters?



20. Inscribe the regular hexagon *ABCDEF* in the circle whose center is *O*. Join the alternate vertices, making the triangle *BFD*. How many degrees are there in each arc subtended by the side of the hexagon? How many degrees are there in each arc subtended by a side of the triangle *BFD*?



The triangle *BCD* is what fractional part of the rhombus *BCDO*? The triangle *BFD* is what fractional part of the hexagon? The line *FH* is what fractional part of the line *FC*? Considering *BD* the base of the triangle *BFD*, what is its altitude? What is the ratio of the altitude of an equilateral triangle to the diameter of the circle in which it is inscribed?

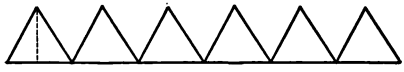
21. What is the length of an arc of 60° in a circle whose radius is 70 inches? How long is the chord which subtends that arc?

22. How long is the perimeter of a segment whose arc is 60° of a circle whose radius is $3\frac{1}{2}$ feet?

23. Find the difference between the straight boundary and the curved boundary of a segment whose arc is 60° of a circle whose radius is 10 feet.

24. If the base of a triangle is 10 feet and its altitude is 8.7 feet nearly, what is the approximate area of the triangle?

25. Inscribe a regular hexagon in a circle. Cut the



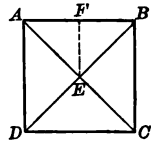
hexagon into triangles. Show that the area of the hexagon is equal to one half the sum of all its sides multiplied by its apothem.

PRINCIPLE 40. — *The area of a regular polygon is equal to one half the perimeter multiplied by the apothem.*

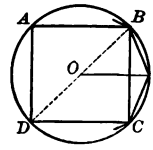
26. A side of a regular hexagon is 8 inches and the apothem is 6.9 + inches. Find the approximate area.

27. A side of a regular pentagon is 6 inches and the apothem is 4.12 + inches. Find the area.

28. Find the area of the square *ABCD* each of whose sides is 10 inches, by dividing it into triangles. How long is the apothem *EF*?



29. With a radius equal to one half the diagonal of a square, describe a circumference. Show that the square is inscribed within it. Bisect each side of the square by a radius and join the extremity of each radius with the adjoining vertices of the square. What kind of a polygon is formed?



30. A side of a regular octagon being 7 inches and the apothem 8.44 + inches, what is the area?

31. Inscribe a regular hexagon in a circle, bisect each arc, and join the points of bisection with the vertices of the adjoining angles of the hexagon. What kind of a polygon is thus formed?

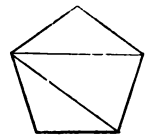
32. The side of a regular dodecagon is 11 inches and the apothem is 20.52 + inches. Find its area.

33. Show how you inscribe a regular dodecagon in a circle.

34. A side of a regular octagon is 9 inches and the apothem is 10.86 + inches. Find its area.

35. Show how you inscribe a regular octagon in a circle.

36. Draw a pentagon, as in the figure, and draw as many diagonals as possible from one of the vertices. Into how many triangles is the pentagon divided? What is the sum of all the angles of the triangles? Then how many degrees have all the angles of the pentagon?



37. Divide a hexagon in the same way and show how many degrees there are in the sum of all its angles.

38. Discover how many degrees there are in the sum of all the angles of a heptagon.

39. Discover the sum of all the angles in an octagon.

40. Have you found out the law by which we can compute the sum of all the angles of a polygon of any number of sides?

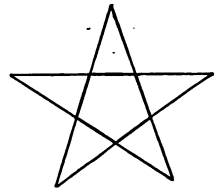
41. How does the number of sides of a polygon compare with the number of triangles into which it is divided by the diagonals drawn from a single vertex?

PRINCIPLE 41. — *The sum of all the angles of a polygon is equal to two right angles taken two less times than the polygon has sides.*

42. How many degrees are there in all the angles of a dodecagon? Of a decagon?

43. How many right angles are there in the sum of the angles of a pentagon? Of a hexagon? Of an octagon?

44. How many degrees are there in each angle of a regular hexagon? Of a regular decagon? Of a regular dodecagon?



45. Find the number of degrees in each angle of the five-pointed star formed by prolonging the sides of a regular pentagon until they meet?

46. Find the number of degrees in each angle of the six-pointed star formed by prolonging the sides of a regular hexagon until they meet.

47. Find the sum of the angles of a polygon of 24 sides.

If n represents the number of sides of a polygon, what represents the number of right angles in the sum of the angles of that polygon?

Find the value of n in the equation $2(n-2) = 36$.

SUGGESTION. — $2(n-2) = 2n - 4$.

48. How many sides has a polygon the sum of whose angles is 36 right angles?

49. How many sides has a polygon the sum of whose angles is 2340°?

50. How many sides has a polygon the sum of whose angles is 1260°?

51. How many sides has the polygon the sum of whose angles is twice the sum of the angles of a hexagon?

52. In what regular polygon is each angle one half as great as an angle of a regular hexagon?

53. Which have the larger angles, regular polygons of few sides or of many sides?

54. Find the ratio of the number of degrees in each angle of a regular hexagon to the number in each angle of a regular dodecagon.

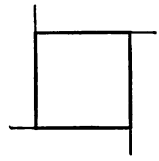
55. Find the ratio of an angle of a regular pentagon to an angle of a regular decagon.

56. Find the ratio of an angle of a regular decagon to an angle of a regular dodecagon.

57. Find the ratio of the sum of the angles of a hexagon to the sum of the angles of an octagon.

58. In what polygon is the sum of the angles three times as great as the sum of the angles of a trapezium?

59. Prolong each side of a square as in the figure. What is the sum of all the exterior angles thus formed?



60. How many degrees has each angle of a regular pentagon? How many degrees has each exterior angle? Find the sum of all the exterior angles.

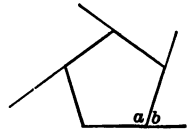
61. Prolong each side of a regular hexagon and discover how many degrees there are in the sum of all its exterior angles.

62. Find in the same way the sum of all the exterior angles of a regular decagon. Of a regular dodecagon.

63. Draw an irregular quadrilateral and find how many degrees there are in its exterior angles.

64. Draw an irregular polygon of three sides and find how many degrees there are in the sum of all its exterior angles.

65. How many degrees are there in the sum of the interior angle *a* and the exterior angle *b*? What is the sum of each pair of adjacent exterior and interior angles of a pentagon? Of a hexagon? Of an octagon?



66. If we let n = the number of sides of a polygon, is the following reasoning correct?

Interior angles $= 180n° - 360°$. (Prin. 41.)

Transposing, we have

Interior angles $+ 360° = 180n°$.

Interior angles + exterior angles $= 180n°$. (Prin. 8.)

Since if either the exterior angles or 360° is added to the interior angles the result is the same, $180n°$, the exterior angles must equal 360°.

PRINCIPLE 42. — *The sum of all the exterior angles of a polygon is equal to four right angles.*

67. What is the ratio of the sum of the interior angles of an octagon to the sum of its exterior angles?

68. Find the ratio of the sum of the exterior angles of a decagon to the sum of its interior angles.

69. Find the ratio of an exterior angle of a regular dodecagon to its adjacent interior angle.

70. What polygon has the sum of its exterior angles equal to the sum of its interior angles?

71. Draw two polygons differing in shape in which each of the interior angles is equal to the adjacent exterior angle.

72. What polygon has the sum of its interior angles equal to twice the sum of its exterior angles?

73. What polygon has each of its interior angles equal to twice the adjacent exterior angle?

74. What polygon has the sum of its exterior angles equal to twice the sum of its interior angles?

75. Of what polygon is each of the exterior angles equal to twice the adjacent interior angle?

76. Of what polygon is each exterior angle equal to two thirds of its adjacent interior angle?

77. Prolong the sides of a regular octagon until they meet, forming a star. Find the sum of all the angles in the points of the star.

78. Can you make a star of a square in the same way? Give reasons.

79. Are all circles similar figures? Are all squares similar figures? Are all rectangles similar figures?

NOTE. — Figures of the same shape are said to be **Similar.**

80. Draw a rectangle 10 centimeters long and 4 centimeters wide, and another 5 centimeters long and 2 centimeters wide. Are their homologous sides proportional, that is, is the ratio of their widths equal to the ratio of their lengths? Are the rectangles similar?

NOTE. — **Similar Polygons** are those whose homologous angles are equal, and whose homologous sides are proportional.

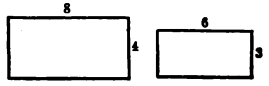
81. Draw a rectangle 4 inches by 3 inches, and another rectangle 12 inches long and of such a width that the homologous sides of the two rectangles shall be in proportion, and show that the rectangles are similar.

82. Draw a rectangle 8 inches by 6 inches, and another 12 inches by 7 inches. Are they similar? If not, change their dimensions so as to make them similar.

83. Construct two similar triangles by making the homologous angles equal and the including sides proportional.

84. Show that the perimeter of a rectangle, 20 millimeters long and 5 millimeters wide, has the same ratio to the perimeter of one 80 millimeters long and 20 millimeters wide, as any two homologous sides.

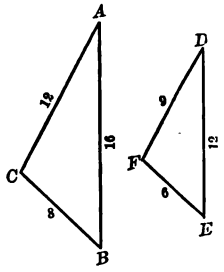
85. Given the rectangles; find the perimeters and show that their ratio is the same as that of the longer sides. Is it the same as that of the shorter sides? Are the rectangles similar?



PRINCIPLE 43. — *The perimeters of similar polygons are to each other as any two homologous sides.*

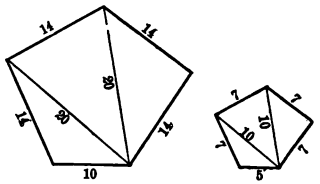
86. Given the similar triangles ABC and DEF; substitute the numbers, and show that the following proportions are true:

$$AB : DE = BC : EF,$$
$$AB : DE = AC : DF,$$
$$BC : EF = AC : DF,$$
$$AB + BC + AC : DE + EF + DF = AB : DE.$$



Show that the perimeters of the triangles are in the same ratio as BC and EF; also in the same ratio as AC and DF.

87. All similar polygons can be divided by diagonals into an equal number of similar triangles placed similarly. Reproduce the given pentagons by constructing triangles of the given dimensions, the units of length being millimeters or centimeters. Show that the perimeters of the pentagons are proportional to any pair of homologous lines.



88. Draw two rectangles whose homologous sides are in the ratio of 3 to 1, and show that their perimeters are in the same ratio. Are their angles equal? Are the rectangles similar?

89. Construct two triangles whose homologous sides are in the ratio of 2 to 3, and show that their perimeters are in the same ratio.

90. The perimeter of an irregular pentagon, one of whose sides is 6 inches, is 33 inches. Find the homologous side of a similar pentagon whose perimeter is 60 inches.

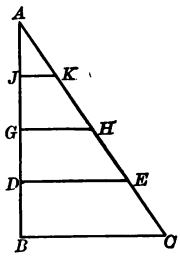
91. Two homologous sides of two similar octagons are respectively 5 centimeters and 9 centimeters. The perimeter of the less octagon is 35 centimeters. Find the perimeter of the greater.

92. The sides of a pentagon are 5, 6, 7, 8, and 9 inches. Find the perimeter of a similar pentagon the shortest side of which is 10 inches.

93. The longest side of a heptagon is 10 inches, and its perimeter is 24 inches. Find the perimeter of a similar heptagon whose longest side is 8 inches.

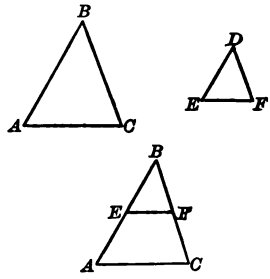
94. The sum of the bases of the greater of two similar isosceles trapezoids is 3 feet, and the lower base is twice the upper. Each leg is $\frac{3}{4}$ of the upper base. The greater base of the less trapezoid is 8 inches. How many inches are there in the perimeter of the less trapezoid?

95. AB is divided into four equal parts by the lines drawn parallel to BC. Find four triangles whose homologous angles are equal. If the perimeter of the triangle AJK is 12 inches, find the perimeter of the others.

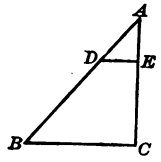


NOTE. — Triangles which are mutually equiangular are similar. How does a line parallel to the base of a triangle divide the remaining sides?

96. If the triangle DEF is superposed upon the similar triangle BAC, can the angle at D be made to coincide with the angle at B? Why? BE being placed upon the homologous side BA, will angle BEF be equal to angle BAC? Why? Is angle BFE equal to angle BCA? If the angles are equal, will the line FE diverge from BC in the same direction as that in which CA diverges from BC, or will it be parallel to CA? If BA is 10 inches, BC 9 inches, BE 5 inches, how long is BF?



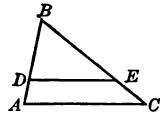
97. DE is parallel to BC. Prove by quoting a geometric principle that angle ADE = angle ABC, and angle AED = angle ACB. Prove by quoting that $AD : DB = AE : AC$. Substitute numbers for the terms of this proportion and show that $AB : AD = AC : AE$.



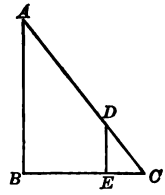
PRINCIPLE 44. — *A line drawn parallel to one side of a triangle forms with the other two sides a triangle similar to the given triangle.*

98. DE is parallel to AC.

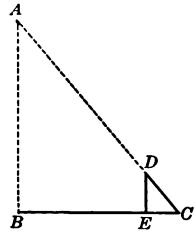
$AB = 7$ inches, $AC = 10$ inches, $BC = 11$ inches, $DB = 5$ inches. Find DA, DE, and BE.



99. AB and DE are both perpendicular to BC. $BC = 10$ feet, $EC = 3$ feet. $DE = 4$ feet. Find AB.



100. A is an inaccessible point directly above B. BC is 20 feet. DE is a stick 5 feet high placed so that its top when sighted from C is in a line with A, and its base is 4 feet from C. Find the height of A.

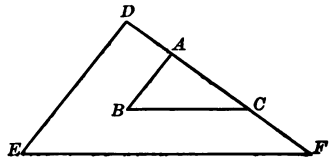


101. When two triangles are similar, can the smaller always be applied to the greater in such a way that they coincide throughout the whole extent of the smaller. Why? Prove from the definition of similar triangles that the homologous sides which do not coincide are parallel.

102. Triangle ABC is similar to triangle DEF.

$AB = 3$ inches. $DE = 7$ inches. $DA = 2\frac{1}{3}$ inches. $AC = 4$ inches. $BC = 5$ inches.

Find CF. Find the perimeter of each triangle.



103. What is the height of a tree whose shadow is 40 feet at the same time that a stick 5 feet high casts a shadow 4 feet in length?

104. Mary and Anna are facing towards the east. Mary, who is 4 feet 6 inches tall, casts a shadow before her 2 feet 3 inches long. How tall is Anna, whose shadow is 2 feet 6 inches? In which part of the day can the conditions of this problem actually occur, forenoon or afternoon?

105. Define all the terms whose meaning has been given in the notes of this chapter.

CHAPTER XII.

CIRCLES AND LINES.

1. How many circles can there be which have a given point as a center?

NOTE. — Circles having the same center are **Concentric.** When they are equal and have the same center, they are **Coincident.**

2. Can you distinguish by the eye each of two coincident circles?

3. Draw two concentric circles and name the figure which is included between their circumferences.

4. Place a circle whose radius is 5 inches inside a circle whose radius is 7 inches so that their circumferences touch, and find the distance between their centers. Find its ratio to the sum of the radii. To the difference of the radii.

NOTE. — When one circle is placed within another in such a way that they are not concentric, they are **Eccentric,** and if their circumferences touch they are said to be **Internally Tangent.**

5. Can you draw two eccentric circles that are not tangent internally?

6. Where have you seen wheels and rods that describe eccentric circles? Describe their working.

7. Draw a circle and place within it several other circles which are tangent to it, making the inner circles equal to one another. From the center of the containing circle,

with a radius equal to the distance from this center to the center of one of the inner circles, describe a circumference. Will it pass through the centers of all the inner circles? Defend your answer.

8. Draw a circle with a radius of 8 centimeters and one with a radius of 7 centimeters, making them externally tangent, and find the distance between their centers. Compare it with the sum of their radii.

9. Draw two circles which are tangent externally, the radius of one being 7 inches and of the other 5 inches. What is the distance between their centers? What is the ratio of the distance between their centers to the sum of their diameters? To the sum of their radii?

10. Draw a circle and several other circles equal to each other, externally tangent to it. Can you draw a circle whose circumference will include the centers of all the external circles?

11. With a radius of 3 inches draw a circle. With a radius of 1 inch draw five circles externally tangent to the first. Draw a line from the center of the inner circle through the center of each outer circle terminating in its circumference. Find the length of each of these lines. Draw a circumference through the terminal points of the lines and find its length. Find the length of the circumference which passes through the center of the outer circles.

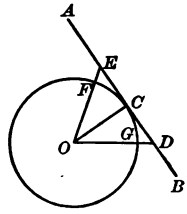
12. Where have you seen wheels which were externally tangent? Describe their working.

13. Draw two circles which intersect, and show that the distance between their centers is less than the sum of their radii.

14. Draw one circle within another and not tangent, and show which is greater, the distance between their centers or the difference of their radii.

15. Draw two circles wholly external, and show which is greater, the distance between their centers or the sum of their radii.

16. AB is a tangent. OC is the radius at the point of tangency. OE and OD are other lines drawn from the center to the tangent. F and G are the points where they cut the circumference. Which is the greater, OE or OF? OF or OC? Why? OE or OC? OD or OC? What is the shortest line that can be drawn from a point to a line?

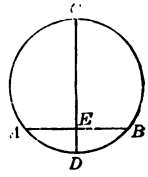


PRINCIPLE 45. — *A tangent to a circle is perpendicular to the radius at the point of tangency.*

17. Referring to Ex. 16, if angle DOC is 40°, how many degrees are there in each angle of the triangle COD?

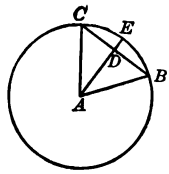
18. Cut out a circle, fold it, and show that a diameter bisects the circle and also the circumference.

19. CD is a diameter perpendicular to the chord AB at the point E. Reproduce the figure, and fold it so that the semicircle CAD is superposed upon the semicircle CBD. Since the diameter bisects the circle, will the line AB be bisected? Will the arc AD equal the arc DB? Will the arc AC equal the arc CB? Show the truth of your answer.



PRINCIPLE 46. — *A radius perpendicular to a chord bisects the chord and the arc subtended by it.*

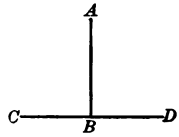
20. AD, 4 centimeters, is perpendicular to CB, 6 centimeters. AE is 5 centimeters. A is the center of the circle. Find the sum of the perimeters of the triangles ADC and ADB.



21. Draw a circle, a chord, a radius perpendicular to the chord, a tangent at the extremity of the radius, and radii to the extremities of the chord. Prolong the radii until they meet the tangent, and show that four similar triangles are formed by the lines.

22. Referring to Ex. 20, if angle CAB is 80°, how many degrees has each angle of the triangle ADC? Of ADB? Quote principles.

23. AB is perpendicular to CD at its middle point. If the distance AC is 10 inches, how long is AD? Quote a geometric principle which is applicable.



24. Take any three points not in a straight line, draw two lines connecting one of the points with the two others, and at the middle points of those lines draw perpendiculars. Will they meet on either side of the lines?

25. Draw perpendiculars at the middle points of the lines AB and BC, which form an angle. From the point O, where the perpendiculars meet, draw lines to A, B, and C. Prove OA, OB, and OC equal.

26. With AO as a radius, draw a circle. Will the circumference pass through B and C? Give reason.

27. Draw a circumference through the three points A, B, and C.

28. Can you place three points so that a circumference cannot be made to pass through them?

PRINCIPLE 47. — *Through any three points not in a straight line a circumference may be made to pass.*

29. Take three points not in a straight line and pass a circumference through them. Can you pass more than one circumference through them? Give your argument.

30. Take two points and see how many circumferences can be made to pass through them.

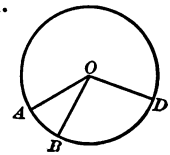
31. How many circumferences may be drawn through a given point?

32. Draw an isosceles triangle and pass a circumference through the vertices of its angles. Supply the missing words in the statements — The circle is _____ about the triangle. The triangle is _____ in the circle.

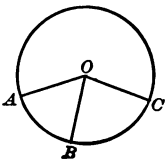
33. How many isosceles triangles can be inscribed in a given circle if the equal sides of all the triangles meet at a given point?

34. With a given chord as a base, how many isosceles triangles can be inscribed in a given circle?

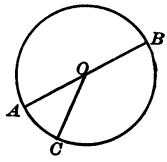
35. Quote the geometric principle which tells how angles at the center of a circle are measured.



36. The arc BD is 3 times the arc AB. Arc $AD = 130°$. How many degrees are the central angles AOB and BOD?



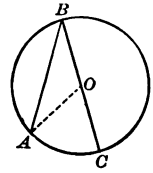
37. Arc AB is 20° less than arc BC in the circle whose center is O. Arc AC is 140°. Find the number of degrees in angle AOB.



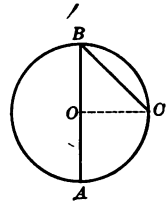
38. Arc BC is $3\frac{1}{2}$ times arc AC. AB is a diameter, and O is its middle point. How many degrees are there in angle AOC? In angle BOC?

39. Inscribe an angle in a circle.

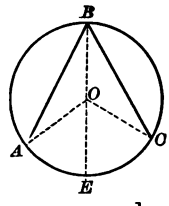
NOTE. — An angle is said to be inscribed in a circle when its vertex is in the circumference and its sides are chords.



40. Angle ABC is inscribed in the circle whose center is O. The arc AC is 60°. We wish to find the number of degrees in the inscribed angle. Draw the construction line AO. How many degrees are there in the exterior angle AOC? How many in the angles at A and B?



41. Angle ABC is inscribed in the circle whose center is O The arc $AC=90°$. How many degrees are there in the angle ABC? What is the ratio of the inscribed angle ABC to the central angle AOC?



42. Angle ABC is inscribed in the circle whose center is O. Arc $AE=50°$. Arc $EC=60°$. How many degrees are there in the angle ABC?

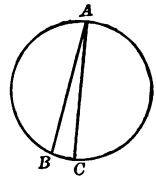
SUGGESTION. — Draw the diameter BOE, and find the number of degrees in each angle thus formed.

43. If there are 50° in a central angle, how many degrees are there in an inscribed angle that has the same arc?

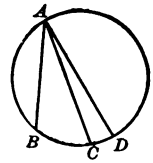
44. We say that "a central angle is measured by the intercepted arc." By what is an inscribed angle measured?

PRINCIPLE 48. — *An inscribed angle is measured by one half the arc intercepted by its sides.*

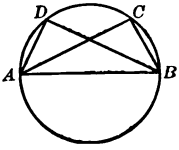
45. Arc AB is 8 times the arc BC. AC is a diameter. How many degrees are there in the angle BAC? How long is the arc BC, if AC is $3\frac{1}{2}$ feet?



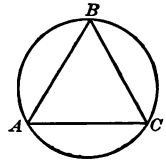
46. Arc AB is 60° greater than arc BC. Arc BC is 30° greater than arc CD. AD is a diameter. How many degrees are there in the angle BAC? How many in the angle CAD?



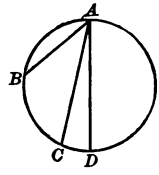
47. AB is a diameter. How many degrees are there in the angle ACB? In the angle ADB?



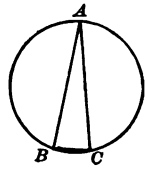
48. The three chords AB, BC, and AC are equal. How many degrees are there in each angle of the triangle which they form? What kind of a triangle is it?



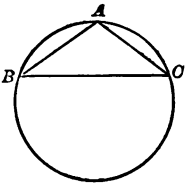
49. The angle BAC is 3 times the angle CAD. AD is a diameter. Arc $BA = 80°$. How many degrees are there in the angle BAC? How many in the angle CAD?



50. The vertical angle A of the isosceles triangle ABC is 30°. Find the number of degrees in each of the equal arcs AB and AC. How long is each arc if the diameter of the circle is 8 feet 2 inches?

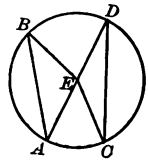


51. The vertical angle A of the isosceles triangle ABC is 3 times a base angle. How many degrees are there in each of the arcs into which the circumference is divided? If angle B were 40° less than angle A, how many degrees would each arc be?



52. How many degrees are there in each angle inscribed in a semicircle?

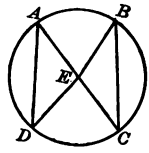
53. The arc AC is 50°. The central angle AEB is 105°. AD is a diameter. How many degrees are there in each angle of the triangle AEB? How many in each angle of the triangle DEC?



54. Arc $AB=70°$.

Arc $DC=80°$. AC is a diameter, and E its middle point.

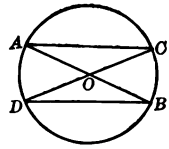
How many degrees are there in each angle of the triangles AED and BEC?



55. AB and CD are diameters.

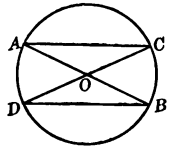
Arc $AD=48°$.

Find each angle of the triangles AOC and DOB.

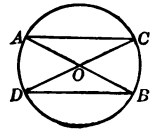


QUERY. — Can diameters intersect at any other point than the center of the circle?

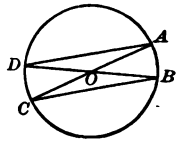
56. Arc $DB=130°$. Find each angle of the triangles AOC and DOB, DC and AB being diameters.



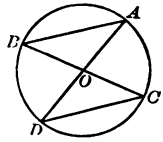
57. Arc DB, which equals arc AC, is 70° greater than the arc AD or its equal BC. Find each angle of the triangles AOC and DOB.



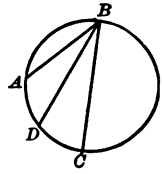
58. Arc BC is 5 times the arc AB. Find all the angles of the triangles AOD and BOC, O being the intersection of the diameters AC and DB.



59. The diameters AD and BC are each 10 inches. The chord AB is 8 inches. Find the perimeter of each triangle. Quote authority.

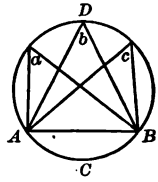


60. The arc DC is 3° greater than the arc AD, and the arc AB is 87°. How many degrees are each of the angles ABD and DBC?



61. The arc ACB is 108°. Which is the greater angle, a, b, or c?

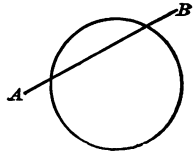
62. Given a line as the hypotenuse. Can you construct a right triangle? Can you construct more than one right triangle having the same hypotenuse?



63. Are all the right triangles which can be constructed on a given hypotenuse equal in area? Argue the point.

64. How long is the radius of a circle which is circumscribed about a right triangle whose hypotenuse is 15 inches?

65. The line AB drawn across a circle cuts the circumference into two arcs, one of which is 140° more than the other. Find each.



NOTE. — A line which cuts a circumference in two points is called a **Secant.**

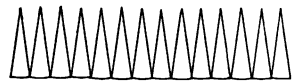
66. Inscribe a regular hexagon in a circle. Bisect each side by a radius of the circle. Are the arcs bisected? Quote authority for your answer. Join the points of bisection of the arcs to the extremities of the chord. Name the polygon formed by the joining lines. If the arcs subtended by the sides of the new polygon were bisected and the points of bisection joined with the extremities of the sides, a polygon of how many sides would be formed?

67. How do you find the area of a regular polygon?

68. Draw a circle and inscribe a square by drawing two diameters at right angles and joining their extremities. Inscribe a regular polygon of double the number of sides by joining the extremities of each side of the polygon with the middle point of the arc subtended by the side. Double the number of sides again, and so on, until the sides are so small that you cannot distinguish the perimeter of the polygon from the circumference of the circle. How would you find the area of the last polygon that you drew, if you knew its side and apothem?

69. If the number of sides of a regular polygon were indefinitely increased so that the polygon would coincide with the circle, to what would the perimeter of the polygon be equal? What would the apothem of the polygon equal?

70. Cut a circle into small sectors. Place them as in the figure. The sum of their bases equals what? Their altitude equals what?



71. Considering a circle as a polygon of an infinite number of sides, the circumference as the sum of those sides and the radius as the apothem, can you see the truth of the following theorem?

PRINCIPLE 49. — *The area of a circle is equal to one half the product of its circumference and radius.*

72. What is the area of a circle whose radius is 14 centimeters?

73. What is the area of a circle whose diameter is 21 millimeters?

74. What is the area of the base of a cone whose circumference is 88 inches?

75. If a horse is tied to a stake on a lawn by a rope 14 feet long, over how many square feet can he graze?

76. What is the area of that part of the face of a watch which is passed over in an hour by a minute hand 16 millimeters long?

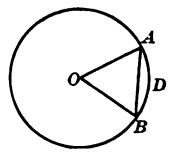
77. What is the area of a sector whose angle is 60° of a circle whose radius is 7 inches?

78. What is the area of a sector which is one fourth of a circle, if its arc is 33 inches long?

79. How far is it from the center of the face of a nickel to its edge? Find the area of both sides of a nickel in square centimeters.

80. If a segment is cut off from a sector which has the same arc, what kind of a figure is left?

81. Angle AOB is 60°. The radius AO is 14 inches, and the altitude of the triangle AOB is 12.12 + inches. Find the area of the sector $AOBD$, of the triangle AOB, and of the segment ABD.



82. A square is inscribed in a circle whose radius is 21 inches. Find the area of each of the segments formed by the sides of the square.

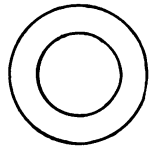
83. A square is circumscribed about a circle whose diameter is 4 feet 8 inches. Find the perimeter of each of the four figures bounded by the arc of a quadrant and one half of each of two adjacent sides of the square. Find the area of each figure.

84. A square whose side is 10 feet is placed within a circle whose radius is $24\frac{1}{2}$ feet. What is the area of the surface between the boundaries of the square and circle? How many vertices of the square can be in the circumference? Illustrate.

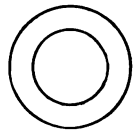
85. A circle $3\frac{1}{2}$ inches in diameter is placed within a square 5 inches in dimensions. Find the area of the surface between the boundaries of the figures.

86. A rectangular garden 20 rods long and 15 rods wide has a circular fountain within it whose circumference is 363 feet. How many square feet of space are given to the fountain, and how much remains?

87. The diameter of the larger circle is $17\frac{1}{2}$ inches, the diameter of the smaller is $10\frac{1}{2}$ inches. Find the area of the circular ring which is the difference between them.



88. The diameter of the large circle is 14 inches. The diameter of the small circle is 8 inches. The area of the small circle is what part of the area of the large circle? The ring is what fractional part of the large circle?



89. What is the area of a sector of 120° of a circle whose radius is $8\frac{3}{4}$ inches?

90. Find the area of a sector of 150° of a circle whose circumference is $12\frac{4}{7}$ centimeters.

91. How many square inches of silk are used for both sides of a fan which when opened is a semicircle, the sticks being 14 inches long, and 7 inches of the sticks next the pivot being uncovered?

92. How many square rods are there in the area of a circular race track, if its outer edge is 1 mile in circumference and its inner edge is 4400 feet in circumference?

93. How many square inches of tin are required to make the bottoms of a dozen pails, the diameter of each being 12 inches?

94. How many square feet are there in the area of two flower beds, each 1320 feet in circumference?

95. Find the areas of two circles, the diameter of one being 3 times that of the other, and show the ratio of their areas.

96. Find the areas of two circles, the diameter of one being 5 times that of the other, and show the truth of the following principle:

PRINCIPLE 50. — *The areas of circles are to each other as the squares of their diameters, or as the squares of their radii.*

97. How many times is the area of a circle whose radius is 2 inches contained in the area of a circle whose diameter is 8 inches? Solve without finding the area of either circle.

98. How many times is the area of a circle whose radius is 3 inches contained in the area of one whose radius is 7 inches?

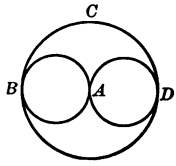
99. The diameter of a circle whose area is 28.57+ square inches is 4 times that of another circle. Find area of smaller circle.

SUGGESTION. — Let x = required area. $4^2 : 1^2 = 28.57 : x$.

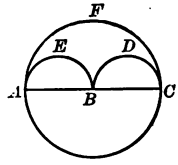
100. The radius of a circle whose area is 41.7 square inches is 3 times that of another circle. Find the area of the smaller circle.

101. The diameter of a round flower bed which holds 1250 plants is 5 times that of a similar one in which the plants are similarly placed. How many plants will the smaller bed hold?

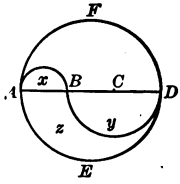
102. A is the center of the circle in which the small circles are placed. Each small circle is what fractional part of the large circle? The irregular figure $BCDA$, formed by the upper semi-circumferences of the three circles is what fractional part of the large circle?



103. AEB and BDC are semicircumferences. The irregular figure $AEBDCF$ is what part of the semicircle ACF?



104. $AB = BC = CD$. The semicircle x is what part of the semicircle ADF? y is what part of ADE? z is what part of ADE? The irregular figure composed of x and z is what part of the whole circle?



105. Find the area of all the surfaces of a cylinder whose bases are 20 centimeters in diameter and whose altitude is 11 centimeters.

CHAPTER XIII.

CUMULATIVE REVIEW NO. 4.

1. When are two lines said to be perpendicular to each other?

2. What is the ratio of an angle of 20° to its supplement? To its complement?

3. Find the exterior angle of a triangle if the opposite interior angles are 20° 35′ and 70° 25′.

4. Is an isosceles triangle a regular polygon? Defend your answer.

5. Find the sum of the perimeters of all the 2-inch squares into which an 8-inch square can be divided.

6. Find the ratio of the sum of the perimeters of all the 5-inch squares into which a 10-inch square can be divided to the perimeter of the 10-inch square.

7. What is the ratio of the area of 3 square inches to that of 3 inches square?

8. What is the ratio of the perimeter of the square which contains 4 square inches to that of the plane figure which is 4 inches square?

9. Find the difference between the area of a rectangle containing 5 square inches, placed side by side, and the area of a plane figure which is 5 inches square.

10. What is the ratio of the perimeter of the figure which contains 6 square inches placed in a row to that of the figure which is 6 inches square?

11. Find the side of a square whose area is equal to that of a rectangle 16 inches long and 4 inches wide. Find the ratio of the perimeters of the square and the rectangle.

12. The side of a square is 18 inches. One of the sides of a rectangle of equal area is 54 inches. Find an adjacent side. Find the difference between the perimeters of the figures.

13. Find the perimeter of a rectangle whose bases are each 32 inches and whose area is equal to that of a square whose side is 24 inches.

14. Find the ratio of the perimeter of a square whose area is 64 inches to the perimeter of a rectangle of equal area a side of which is 32 inches.

15. Find the difference in the cost of fencing a square lot whose side is 12 rods and a rectangular lot of equal area one of whose sides is 36 rods, the fence costing 10 cents a foot.

16. Find the difference in the cost of the outside foundation walls of a house which contains four rooms, each 16 feet square, placed in a row, and a square house containing four rooms each 16 feet square, if each lineal foot of the foundation costs 50 cents and no allowance is made for partitions or corners.

17. What will be the difference in the cost of the flooring of the houses if each square foot costs $18\frac{3}{4}$ cents?

18. Given a square, and a rectangle of equal area which is not a square. Which has the greater perimeter? Argue the case.

19. Of rectangles having equal perimeters, which has the greater area, a square or a rectangle which is not a square? Illustrate.

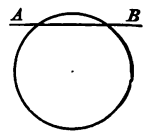
20. Draw a rhomboid and a rectangle having equal bases and perimeters and show which has the greater area.

21. Find the difference in area between a square whose boundary is 88 inches and a circle whose boundary is 88 inches.

22. When Queen Dido, landing on the shores of Africa, wished to inclose the greatest amount of land by the strips of hide, should she have laid them so as to inclose a square or a circle?

SUGGESTION. — See encyclopedia or classical dictionary for the story of Queen Dido.

23. The line AB cuts the circumference into two arcs, one of which is 4 times the other. How many degrees are there in each arc? What name is given to a line which cuts a circumference in two points?

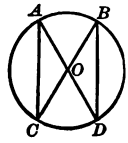


24. Draw a circle and two secants meeting at a point without the circle.

25. Draw a circle, a chord, and radii to the extremities of the chord. Join the middle point of the chord with the center of the circle. What kind of triangles are thus formed? Quote geometric principle.

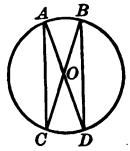
26. AC is parallel and equal to BD.

Arc $CD = 60°$. Find all the angles of the triangles AOC and BOD.



27. O is the middle point of the diameters AD and BC. Arc BD equals 4 times the arc AB.

Find each angle of the triangles COA and BOD.



28. Show how you circumscribe a circle about a scalene triangle.

29. Given a circle. Inscribe a scalene triangle. An isosceles triangle.

30. In the triangle ABC angle A = two times angle B and angle C is 33°. How many degrees are there in angle A and in angle B?

31. Find the sum of the exterior angles made by prolonging the equal sides of an isosceles triangle whose vertical angle is 45°.

32. Can you draw an isosceles right triangle? Can you draw an isosceles triangle having its base angles right angles? Can you draw an equilateral right triangle?

33. Can you divide an equilateral triangle into equal triangles by one line? An isosceles triangle? A scalene triangle?

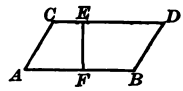
34. Can you divide a scalene triangle into two triangles which are equivalent?

NOTE. — Observe the distinction between equal and equivalent. Equal polygons are those which can be made to coincide in all their parts. Equivalent polygons are those which have the same area.

35. Can a square be equal to a rhomboid? Can it be equivalent to a rhomboid?

36. $ABCD$ is a rhomboid.

$AB = 9$ inches. EF, the altitude, $= 4$ inches. Find the side of a square of equal area. What is the ratio of the perimeters of the figures?



37. The area of a rhomboid is 240 square inches and one of the sides is 2 feet. Find the altitude perpendicular to that side.

38. The area of a rhomboid is 270 square feet and one of the altitudes is 10 feet. Find the corresponding base.

39. The side AD of the trapezium $ABCD$ is 3 inches longer than the side DC. DC is 2 inches longer than BC, BC equals AB, and the perimeter is 27 inches. Find each side.

40. Can you draw a trapezoid having a right angle at each extremity of one of the non-parallel sides?

41. Can you draw a trapezoid having a right angle at each extremity of a base?

42. Find the angles of a rhombus each of whose obtuse angles is three times a consecutive angle.

43. Find the perimeter of the regular hexagon inscribed in a circle whose radius is 9 centimeters.

44. A regular hexagon is inscribed in a circle whose radius is $31\frac{1}{2}$ centimeters. Find the sum of the perimeters of the three equal rhombuses into which the hexagon can be divided by three radii.

45. Inscribe a regular hexagon in a circle and join the alternate vertices. What figure is formed? What is its ratio to the hexagon?

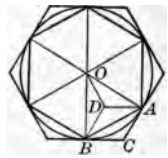
46. Show how you inscribe an equilateral triangle in a circle.

47. Can you divide a regular hexagon into twelve equal scalene triangles?

48. Can you divide a regular hexagon into six equal isosceles triangles?

49. Can you divide a regular hexagon into six equal equilateral triangles?

50. Inscribe a hexagon in a circle. Draw radii to the vertices of the hexagon, draw tangents perpendicular to those radii, and prolong them until they meet. You have circumscribed a hexagon about a circle. Fold the outer triangles, as the triangle *ABC*, upon the hexagon and discover what part an inscribed hexagon is of a circumscribed hexagon.



QUERY. — How do the triangles *ADO*, *BOD*, and *ABD* compare, *D* being the middle point of the triangle *OAB*?

51. If any point on a circumference is joined with the extremities of a diameter, what kind of a triangle is formed?

52. Find the ratio of the areas of two circles, the diameter of one being 1 decimeter and of the other 15 centimeters.

53. Find the ratio of the areas of two circles, the diameter of one being 5 meters and the radius of the other being 5 decimeters.

54. Find the area of an isosceles triangle whose base is 5 decimeters and altitude 10 centimeters. Find the area of a scalene triangle having the same base and altitude, and compare the areas of the triangles.

55. Reproduce the right-angled scalene triangle ABC, whose base is 4 centimeters and altitude 7 centimeters, and construct an isosceles triangle having the same base and area.

56. Find the fourth proportional to three lines respectively 6 inches, 8 inches, and 12 inches.

57. What line has the same ratio to a line 16 inches long that a line 5 inches long has to one 8 inches long?

58. How long is a line whose ratio to a line 20 inches long is the same as that of a line 7 inches long to a line 28 inches long?

59. Find the ratio of the area of a rectangle 9 inches long, 5 inches wide to one 10 inches long, 3 inches wide.

60. A rectangle 9 inches long, 7 inches wide is in the same ratio to one 7 inches long, 4 inches wide as a square 9 inches in dimensions is to another square. Find the perimeter of the smaller square.

61. What is the ratio of one of the angles of a regular decagon to the sum of its exterior angles?

62. A semicircle is divided into two sectors, the arc of one of which is 30° more than that of the other. The greater sector is what part of the semicircle?

63. Which is nearer the center of the circle, a chord 8 centimeters long or one 6 centimeters long in the same circle? Illustrate and quote geometric principle.

64. Show how you inscribe a square in a circle.

65. Draw a circle and two diameters at right angles. At each extremity of each diameter draw a parallel to the other diameter, prolonging them until a surface is inclosed. What names belong to the polygon thus formed by the four tangents?

66. What is the area of the square circumscribed about a circle whose radius is 12 inches?

67. What is the ratio of a square circumscribed about a circle to a square inscribed in the same circle?

68. The end of a round log is 12 inches in diameter. What is the area of the end of the largest square stick of timber that can be cut from it?

69. How many times will a wheel whose circumference is 25 inches revolve in traveling 30 feet?

70. How many times will a wheel whose diameter is 63 inches revolve in traveling 115 feet 6 inches?

71. The base of an isosceles triangle whose perimeter is 80 inches is 20 inches less than twice one of the equal sides. Find the length of the greatest common measure of the sides of the triangle. How many times can it be laid off on the perimeter?

72. Bisect two angles of an equilateral triangle, and find how many degrees there are in the angles formed by the bisectors. How many degrees are in the angles formed by the bisectors of the base angles of an isosceles triangle whose base angles are each twice the vertical angle?

73. The perimeter of an isosceles triangle whose base is its shortest side is 100 centimeters. The difference

between the base and an adjacent side is 23 centimeters. The altitude is 40 centimeters. Find the area.

74. The difference of angle a and angle b is 12°. Find each angle of the rhomboid.

75. The longest side of a trapezium whose perimeter is 49 inches is 21 inches. Find the longest side of a similar trapezium whose perimeter is 35 inches, and quote the geometric principle upon which your work depends.

76. Include an angle of 120° by two lines each 5 centimeters. At the end of one of the lines make on the same side another angle of 120° with a line 5 centimeters. Continue this operation until you have inclosed a surface, and write its name upon it.

77. Take Ex. 76, substituting 135° for 120°.

78. Construct a regular hexagon whose sides are each 5 centimeters. Its apothem is 4.33+ centimeters. What is its area?

NOTE. — If the hexagon were to rise in the air, keeping itself always parallel to its first position, the geometric solid which its path would form would be a **Hexagonal Prism.**

79. If the hexagon in Ex. 78 rose to the height of 10 inches, what would be the area of all the surfaces of the hexagonal prism?

80. What is a prism? What are its bases?

SUGGESTION. — See dictionary.

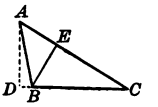
81. Find the lateral surface of a triangular prism, each side of the bases being 3 feet and the height of the prism being 9 feet.

82. Find all the surfaces of a regular pentagonal prism, one side of a base being 8 feet, the apothem being 5.49+ feet, and the altitude of the prism 12 feet.

83. What is the height of a flag pole whose shadow is 5 feet when the shadow of an upright stick 4 feet long is 3 inches?

84. The length of a stick and the length of its shadow when placed upright are equal. At the same time what is the length of the shadow of a steeple 48 feet high?

85. The line AD, which shows the altitude of the triangle ABC, when BC is considered the base, is 4 inches. BC is 6 inches, BE, the altitude, when AC is considered the base, is 3 inches. Find AC.



86. Arrange the following in three lists, one of lines, one of surfaces, the other of solids: Chord, circle, hexagon, octagon, cube, segment, arc, quadrilateral, sector, trapezium, pyramid, polygon, cone, perimeter, diameter, sphere, semicircle, circumference, radius, rectangle, quadrant, prism, liter, triangle, measure of an arc, measure of a sector, cylinder, semicircumference, decagon, rhombus, measure of a line, trapezoid, plane, diagonal, rhomboid, transversal, dodecagon, secant, median, square, parallelogram, tangent, parallelopiped.

CHAPTER XIV.

SQUARES AND CUBES.

1. Complete the table of the squares of the integers from 1 to 25 inclusive. $1^2 = 1$, $2^2 = 4$, etc.

Make a table of the squares of numbers expressed by a significant figure and a cipher, as 10, 20, 30, and show how the second table is derived from the first.

2. Find with the help of your tables the side of a square whose area is 484 square meters. Of one whose area is 48,400 square meters.

3. Find the side of a square whose area is 529 square feet. Of one whose area is 52,900 square feet.

4. Find the length, width, and area of the rectangle formed by placing two squares, each containing 361 square inches, so that they have a common side. Illustrate.

5. Find the length of a rectangle twice as long as broad whose area is 1152 square inches.

6. Find the perimeter of a square whose area is 441 square meters.

7. Find the perimeter of a rectangle 3 times as long as broad whose area is 588 square meters.

8. What line squared and multiplied by 3 gives a rectangle containing 768 square inches?

9. If x represents the side of a square, what represents its area? The rectangle which contains three such squares? The width of the rectangle? Its length?

10. Find the width and length of a rectangle which is 5 times as long as wide, and contains 2205 square inches.

11. A rectangular lot whose length is 4 times its width contains 676 square rods. Find its dimensions.

12. How many yards of binding will it take for a square oilcloth mat covering 81 square feet?

13. Of two squares, the greater contains 24 square feet more than the less. The sum of their areas is 74 square feet. Find the side of each.

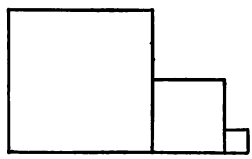
SUGGESTION. — Let x = side of smaller square.
Then x^2 = area of smaller square,
$x^2 + 24$ = area of greater square.

14. The first of four squares contains 63 square inches more than the second, the second 17 square inches more than the third, the third 28 square inches more than the fourth. Their total area is 325 square inches. Find the side of each.

15. There are three squares, the first of which is 4 times the second, and the second is 9 times the third. Their combined area is 414 square feet. Find the length of their combined perimeters.

16. Find the number of rods of fencing required to inclose separately three square lots, the first of which contains 11 square rods more than the second, and the second contains 16 square rods more than the third, their combined area being 70 square rods.

17. Three squares are arranged as in the diagram. They cover 184 square inches.



Find the length of the boundary line of the surface covered by them if the square on the left is 4 times as great as the middle

square, and the middle square is 9 times as great as the one on the right.

18. The perimeter of a certain square is 4 inches longer than that of another square, and the sum of their perimeters is 100 inches. Find the sum of their squares.

19. How many times is the square of any number contained in the square of twice that number? Illustrate.

20. Draw squares, and show how many times the square of a 4-inch line is contained in the square of an 8-inch. In the square of a 12-inch line.

21. How many times is the square of any number contained in the square of 3 times that number?

22. How many times does the square of a line contain the square of $\frac{1}{3}$ of that line?

23. A square inch can be divided into how many figures each $\frac{1}{4}$ of an inch square?

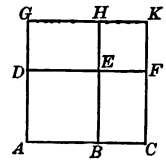
24. Draw a figure which is the sum of the squares of two lines, one 8 inches, the other 3 inches, and find its area.

25. Draw a figure which is the square of the sum of the same lines, and find its area.

26. Find the difference between the sum of the squares and the square of the sum of two lines, one 7 inches, the other 8 inches.

27. Draw a rectangle which is the product of two lines respectively 8 inches and 5 inches, and find its area and perimeter.

28. Let AB and BC be two lines respectively 6 inches and 4 inches, and AC their sum.



How many square inches are there in the square *ADEB*?

How many square inches are there in the square *EHKF*?

How many square inches are there in the rectangle *BEFC*?

How many square inches are there in the rectangle *DGHE*?

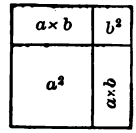
How many in all?

PRINCIPLE 51. — *The square of the sum of two lines is equal to the square of the first plus twice the product of the first by the second plus the square of the second.*

29. Draw the square of the sum of two lines, one 5 inches, the other 3 inches, and show the truth of Prin. 51.

30. Square the sum of a and b, a representing a line 7 inches, and b representing a line 2 inches.

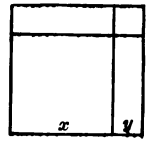
31. Draw a figure, and show that $(a+b)^2 = a^2 + 2(a \times b) + b^2$ when $a =$ a 10-inch line and $b =$ a 3-inch line.



32. Let a and b be two lines. Show by numbers that the square of their sum is represented by the diagram.

33. $x = 10$ inches, $y =$ a line less than 10 inches, and the square of their sum $= 289$ square inches. Find y.

34. $x = 10$, $y =$ a less number, the square of their sum $= 324$. Find y.



35. $x = 10$ inches. Find by trial the value of y, if $(x+y)^2 = 169$.

36. The square of the sum of two lines is 256 square inches; the greater is 10 inches. Find the less.

37. The square of the sum of two lines is 441 square inches; the greater is 20 inches. Find the less.

38. Find the sum of the separate perimeters of the squares and rectangles which make the square of the sum of a line 20 inches and a line 3 inches.

39. If you had 196 square feet of lumber which you wished to place in the form of a square, and were to begin by making a square 10 feet each way, how wide must the additions be in order to complete the square?

40. Find the square root of 1849, and show how Principle 51 assists you in the work. Why do you double the first figure found in the root in order to obtain a trial divisor?

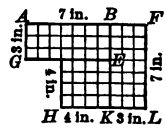
SUGGESTION. — See arithmetic for rule for extracting the square root of numbers.

41. Find the square root of 1225, 2601, 5184, 3844, 5329, 5929, 8649, 12769, 14884, 94249, 495616.

42. Lay off the line AB 4 inches on the line AD 7 inches; draw a square on the difference, and find its area.

43. Construct a square on the difference of the lines 11 inches and 6 inches, and find its area. Find the sum of the squares of those lines.

44. Given a line 7 inches and one 3 inches. How many square inches are there in the square of their difference? In the sum of their squares? How many square inches are there in the rectangle $ABEG$? Which is greater, the rectangle $ABEG$ or the rectangle $KLFB$? Reproduce the figure which is the sum of the squares of the lines, cut out the square of their difference and see what remains.



PRINCIPLE 52. — *The square of the difference of two lines is equal to the sum of their squares minus twice the rectangle of the lines.*

45. Given the lines x, 8 inches and y, 3 inches. Cut out the figure made of the sum of the squares of the two lines, and show by superposing that if the square of the difference of these lines is cut from the figure, the remaining space will be equal to two rectangles of these lines.

Show the same truth about other lines until you realize the meaning of Prin. 52.

46. Draw a figure and show the truth of the statement $(a-b)^2=a^2-2(a\times b)+b^2$, when $a=$ a line 5 inches and $b=$ a line 3 inches.

47. Draw a rectangle which is the product of the sum of two lines by the greater line, the lines being 8 inches and 5 inches, and find its perimeter.

48. Draw a rectangle which is the product of the difference of two lines by the greater, and find its area; the lines being 12 inches and 9 inches.

49. There are two lines 6 inches and 5 inches respectively. Draw the rectangle of the sum of the lines by the less, and find the area and perimeter.

50. Given one line 5 inches, another 3 inches. Draw the rectangle of their sum and difference, and give its area.

51. Draw a rectangle which is the product of the sum and the difference of two lines, one of which is 7 inches and the other 3 inches, and show that the number which represents its area is equal to 7^2-3^2.

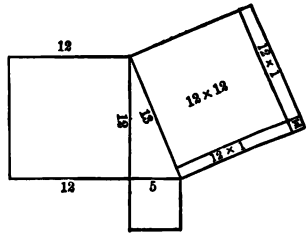
PRINCIPLE 53. — *The rectangle of the sum and difference of two lines is equal to the difference of the squares of the lines.*

52. Draw the rectangle which is the product of the sum and difference of two lines respectively 7 inches and 2 inches. Draw the figure which is the difference of the squares of those lines, the smaller square being subtracted from the upper right-hand corner of the greater. Superpose this figure upon the first, cutting off the rectangle at the top, which is the product of the less line by the difference of the lines, and applying it at the side. Show the application of Prin. 53.

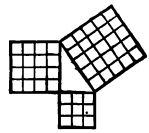
53. Find the area and perimeter of the rectangle of the sum and difference of two lines whose sum is 15 inches, one of which is twice the other.

54. Find the area of the rectangle of the sum and difference of two lines whose sum is 15 inches, one line being 3 inches longer than the other, and show by diagrams how much this rectangle lacks of being equal to the square of the greater line, and how much it exceeds the square of the less.

55. Draw a right triangle having a base 5 inches and perpendicular 12 inches. It will be found that the hypotenuse is 13 inches. Construct a square upon each side, and cut out the figure. Apply the square of the perpendicular to the square of the hypotenuse. Cut and fit the square of the base to the remaining space.



56. Draw a right triangle having a base 3 inches and perpendicular 4 inches. The hypotenuse will be 5 inches. Construct a square upon each side, and apply the squares of the base and perpendicular to that of the hypotenuse.



NOTE. — Demonstrative geometry will show the truth in all cases of the following principle:

PRINCIPLE 54. — *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

57. Measure off 6 inches on one side of a square surface and 8 inches on the adjacent side. How long is the line that joins the extremities of these two lines?

58. A travels 30 miles east from a certain point; B 40 miles north from the same point. How far apart are they?

59. Draw a rectangle 12 centimeters long and 9 centimeters wide, and find its diagonal.

60. What is the distance from the lower left-hand corner to the upper right-hand corner of a gate which is 8 feet long and 6 feet wide?

61. What is the length of the longest stick pointed at both ends that can lie wholly on a table which is 24 feet long and 7 feet wide?

QUERY. — Why must it be pointed?

62. What is the distance diagonally across a rug 16 feet long and 12 feet wide?

63. A vacant lot is 24 rods long and 18 rods wide. Mary crossed it diagonally, and Anna reached the same point by walking along the edge of the lot. How much farther did Anna walk than Mary?

64. The base of an isosceles triangle is 30 feet, and its altitude is 36 feet. Find one of the equal sides.

QUERY. — Into what two equal figures is an isosceles triangle divided by a perpendicular drawn from the vertex to the base?

65. The base of an isosceles triangle is 64 inches, and its altitude is 24 inches. Find its perimeter.

66. The base of an isosceles triangle is 6 inches, and its area is 12 square inches. Find its altitude and one of the equal sides.

67. Find the perimeter of an isosceles triangle whose base is 20 feet, and whose altitude is 24 feet.

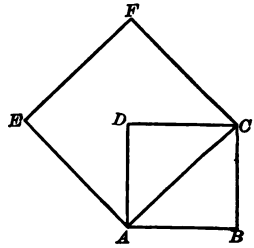
68. The base of a right triangle is 15 centimeters, and the perpendicular 20 centimeters. Their sum is how much more than the hypotenuse?

69. Find the approximate length of the hypotenuse of a right triangle whose base is 12 inches and perpendicular 10 inches, carrying the work to two places of decimals.

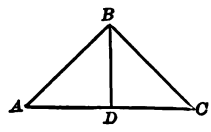
70. Find the diagonal of a square whose side is 8 feet.

71. Can you show from the accompanying figure that the square of the diagonal of a square is twice the original square?

SUGGESTION. — Draw construction lines *DF* and *DE*.



72. *ABC* is an isosceles triangle right angled at *B*, and *BD* is its altitude. Which is the greater, the square of *BD* or the triangle *ABC*? Argue the point.



73. How long a rope is required to reach from the top of a pole 60 feet high, to a point on the ground 80 feet from the center of its base?

74. The diameter of a round cistern is 12 feet, and the water is 5 feet deep. Find the length of the longest stick pointed at both ends that can touch at the same time the bottom of the cistern and the surface of the water.

QUERY. — Can you imagine the cistern and stick?

75. Show that $x^2 = a^2 + b^2$ when a and b are lines respectively 27 inches and 36 inches, including a right angle, and x is a line 45 inches joining their extremities.

76. How can you find the hypotenuse of a right triangle when the base and perpendicular are given?

77. Show that $x^2 = a^2 - b^2$, when a, the hypotenuse of a right triangle, is 39 inches, b, the base, is 15 inches, and x, the perpendicular, is 36 inches.

78. Given the hypotenuse and base of a right triangle, how can you find the perpendicular?

79. Hypotenuse 55 feet, base 44 feet, perpendicular = ?

80. Hypotenuse 39 feet, base 36 feet, perpendicular = ?

81. A ladder 26 feet long is leaning against a house, and its base is 10 feet from the base of the house. How high does it reach?

82. One of the equal sides of an isosceles triangle is 25 inches, and its base is 14 inches. Find its altitude.

83. The perimeter of an isosceles triangle is 144 inches, and its base is 40 inches. Find its altitude and area.

84. The perimeter of an isosceles triangle is 50 feet. Each of the equal sides is 1 foot longer than the base. Find its altitude and area.

85. Given the hypotenuse and perpendicular of a right triangle; how do you find the base?

86. Hypotenuse 75 feet, perpendicular 72 feet, base = ?

87. Hypotenuse 78 feet, perpendicular 72 feet, base = ?

88. What is the perimeter of a right triangle whose perpendicular is 24 centimeters and whose hypotenuse is 25 centimeters?

89. One side of an isosceles triangle is 13 feet, the altitude is 12 feet. Find the base and area of the triangle.

90. The sum of the equal sides of an isosceles triangle is 30 feet, and the altitude is 12 feet. Find the base and area.

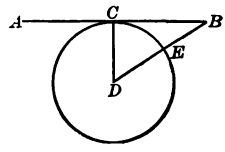
91. How far away from the base of a house must a 20-foot ladder be placed that its top may reach a window 16 feet from the ground?

92. The distance from the center of a circle whose radius is 10 inches to the middle point of a certain chord is 8 inches. How long is the chord?

93. Find the perimeter of a rectangle one side of which is 30 meters, and the diagonal of which is 50 meters.

94. A house is 30 feet wide, and the perpendicular distance from the level of the eaves to the comb of the roof is 8 feet. Find the length of the rafters.

95. *AB*, 18 inches, is tangent at its middle point, *C*, to the circle whose diameter is 12 inches. Find the length of *EB*.



96. One of the legs of an isosceles right triangle is 8 feet. Find the base. Prove that the altitude is one half the base, using the figure in Ex. 72.

97. One of the legs of an isosceles right triangle is 10 feet. Find its altitude and area.

98. How many times is an isosceles right triangle contained in the square of its base?

99. Find the altitude of an equilateral triangle each of whose sides is 10 feet, carrying it to two places of decimals.

100. Find the approximate altitude and area of an equilateral triangle whose side is 8 feet.

101. Find the area of a rhombus whose side is 9 feet and whose shorter diagonal equals a side.

102. Can you show that the diagonals of a rhombus bisect each other at right angles?

SUGGESTION. — See Geom. Prin. 25.

103. The diagonals of a rhombus are 12 inches and 16 inches. Find a side of the rhombus. What is the area of the rhombus?

104. The sum of the diagonals of a rhombus is 34 inches, and the long diagonal is 14 inches longer than the short diagonal. Find the diagonals, perimeter, and area of the rhombus.

105. The long diagonal of a rhombus whose side is 26 centimeters is 48 centimeters. Find the short diagonal.

106. Find the area of an equilateral triangle each side of which is 12 inches.

107. Place six equilateral triangles each of whose sides is 12 centimeters around a common point, and find the approximate area of the hexagon thus formed.

108. Include an angle of 108° by two lines each 6 inches. At the extremity of one of the lines make another angle of 108° on the same side of the line. Continue this process, using 6-inch lines until a surface is inclosed, and write its name.

109. Connect each vertex of a regular pentagon with its center. How many and what kind of triangles are thus formed? Join the middle point of one of the triangles with the center of the pentagon, and write the name of the joining line upon it.

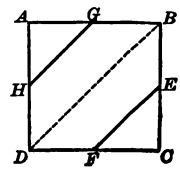
110. Draw a regular octagon whose sides are each 4 inches by the method given in Ex. 108, making angles of 135°. Measure its apothem, and find its approximate area.

111. Draw by the same method a regular dodecagon whose sides are each 4 inches. Measure, and compute area.

112. How long will it take a spider traveling 3 inches a second to cross diagonally a floor 28 feet long and 21 feet wide?

113. Join the middle points of two adjacent sides of a square, and cut along the line thus drawn. What fractional part of the square has been cut off? Prove.

114. Given the square *ABCD*, whose area is 64 square inches; *H*, *G*, *E*, and *F* are the middle points of their respective lines. Find the area of the irregular hexagon *GBEFDH*. Of the trapezoid *DHGB*.



115. In the same figure find the length

of diagonal *DB*. Of line *HG*. Find the perimeter of trapezoid *DHGB*, and of hexagon *GBEFDH*.

116. Find the length of a chord of 90° in a circle whose radius is 6 inches.

117. Find the perimeter of a segment whose arc is 90° in a circle whose radius is 7 inches.

118. Cut a right triangle out of pasteboard and, keeping the perpendicular upright, revolve the triangle. What geometric solid is formed by its path? What surface is formed by the path of the hypotenuse?

119. The altitude of a cone is the perpendicular distance from the vertex to the base. The slant height of a cone is the distance from the vertex to the circumference of the base. If the perpendicular of a cone is 12 inches and the radius of the base is 5 inches, what is its slant height? What is the area of its base?

120. What is the slant height of a cone whose altitude is 24 inches, and whose diameter is 14 inches?

121. What is the length of the longest line that can be drawn on a blackboard 12 feet long and 5 feet wide?

122. How long is the longest stick that can be carried diagonally through a doorway 8 feet high and 6 feet wide?

QUERY. — Must the ends of the stick be sharp pointed?

123. What is the diameter of the largest circular saw that can be carried through a gateway 12 feet wide and 9 feet high?

124. The perpendicular of a right triangle is 9 inches longer than the base. The perimeter is 108 inches, and the hypotenuse is 45 inches. Find the altitude, base, and area of the triangle.

125. A regular hexagon whose side is 5 inches is drawn within a circle whose diameter is $17\frac{1}{2}$ inches. What is the area of the surface between the perimeter of the hexagon and the circumference of the circle? Draw diagram.

126. A regular hexagon is inscribed in a circle whose radius is 28 inches. How many square inches are in the sum of all the segments formed by the sides of the hexagon?

127. An equilateral triangle is inscribed in the same circle. Find the area of the segments cut off by the sides of the triangle.

QUERY. — An inscribed equilateral triangle is what fractional part of a regular hexagon inscribed in the same circle?

128. Find the area of a segment whose arc is 120° in a circle whose radius is 63 inches.

129. Place two squares, one 8 inches, the other 6 inches, in dimensions, so that a right angle is formed by their sides. How would you find the side of the square which is equivalent to their sum?

SUGGESTION. — See Geom. Prin. 54.

130. Find the side of a square which is equivalent to the sum of a square 8 inches and one 6 inches in dimensions.

131. Find the perimeter of a square whose area is equal to that of two squares, one 9 inches, the other 12 inches in dimensions.

132. Define similar polygons.

133. Draw a right triangle whose base is 5 centimeters and perpendicular 3 centimeters. Draw another whose base is to the base of the first triangle as 2 : 1, and whose perpendicular is in the same ratio to the perpendicular of the first triangle. Find the ratio of their areas.

134. Draw right triangles whose homologous sides are in the ratio of 3 to 1, and show by numbers that their areas are to each other as 9 to 1.

135. You have shown that the areas of similar right triangles are to each other as the squares of their homologous sides. In the same way show the ratio of the areas of similar rectangles.

136. Are circles similar figures? Show that the areas of circles are to each other as the squares of their homologous lines.

NOTE. — Demonstrative geometry will show you the truth of the following principle:

PRINCIPLE 55. — *The areas of similar polygons are to each other as the squares of their homologous sides.*

137. The area of a heptagon, one of whose sides is 6 inches long, is 100 square inches. Find the area of a similar heptagon whose corresponding side is 9 inches long.

138. Find the area of a pentagon whose shortest side is 4 centimeters when the area of a similar pentagon whose shortest side is 5 centimeters is known to be 50 square centimeters.

139. A field, one side of which is 20 rods, contains 5 acres. How many acres are there in a similar field, whose corresponding side is 15 rods?

140. Mary dressed two dolls which were exactly similar, but of different sizes. The hem of the apron of the larger doll was 10 inches long and its area was 48 square inches. The hem of the apron of the smaller doll was 5 inches long. What was the area of its apron?

141. The area of a polygon whose longest side is 12 inches is 36 square inches. Find the longest side of a similar polygon whose area is 64 square inches.

SUGGESTION. — Let x = the required side.

142. Complete the table of the cubes of the integers from 1 to 25 inclusive. $1^3=1$, $2^3=8$, etc.

Derive a table of the cubes of 10, 20, etc., from the preceding table.

143. Build a cube of a line 4 inches long.

NOTE. — A cube which has a certain line for each of its edges is said to be the cube of that line.

144. How many faces and how many edges has a cube?

145. How many 1-inch cubes make the first layer of a 4-inch cube? How many are there in all the layers?

146. Give the volume and the superficial area of x^3, x being a 5-inch line.

147. How many 1-inch cubes are there in the lowest layer of a cube which is 8 inches in dimensions? How many layers compose the cube?

148. What is the length of one edge of a cube which contains 2197 cubic inches? Of all its edges?

149. What is the area of one face of a cube which contains 5832 cubic inches? Of all its faces?

150. What is the volume of a cube one face of which contains 81 square centimeters?

151. What is the volume of a cube whose faces contain 294 square decimeters?

152. How long is the side of a cubical vessel containing 343 cubic centimeters?

153. The length, height, and width of a certain room are equal. It contains 729 cubic feet. Find the distance diagonally across the floor of the room.

154. Find the sum of all the edges of a cube which contains 125 cubic centimeters.

155. Find the area of all the faces of a cube which contains 1728 cubic inches.

156. Find the sum of all the diagonals which can be drawn on the faces of a cube containing 216 cubic inches.

157. Find the sum of the perimeters of all the faces of a cube which contains 512 cubic inches.

158. Find the area of all the squares which can be inscribed in the faces of a cube containing 1331 cubic inches.

159. How many cubes, one inch each way, can be cut from a cube 4 inches in its dimensions?

160. How many cubes, each one quarter of an inch in its dimensions, can be cut from an inch cube?

161. How many inch cubes can be cut from a 3-inch cube?

162. How many 2-inch cubes can be cut from a 6-inch cube?

163. How many 3-inch cubes can be cut from a 6-inch cube?

164. Find the edge of the cube which is the G.C.M. of two cubes, one containing 27 cubic inches and the other 216 cubic inches. Find the edge of the smallest cube that will contain each of them a whole number of times.

165. Find the edge of a cube which is the L.C.M. of two cubes, one containing 216 cubic inches, the other 64 cubic inches. Find the largest cube which can be cut from each of them a whole number of times.

166. How many cubic centimeters are there in a liter? How many cubic millimeters are there in a cubic centimeter?

QUERY. — Can you make a cubic millimeter?

167. Find the sum of all the edges of a cube containing 1728 cubic inches.

168. Find the sum of all the edges of 7 cubes, each containing 64 cubic inches.

169. Find the area of all the surfaces of a cube whose contents are 125 cubic inches.

170. Find the area of the surface of a box twice as long as wide, whose width equals its height, and whose contents are 128 cubic inches.

SUGGESTION. — Mold the shape in clay, or make it in pasteboard, or cut it out of a potato or turnip, or, best of all, form a clear mental image of it.

171. Find the area of the surfaces of a box whose width and height are equal, length twice the width, and contents 250 cubic inches.

172. Give the volume of a parallelopiped 5 inches long, 4 inches wide, and 3 inches high, and the area of its surfaces.

173. Find the cubic contents and area of inside surfaces of a box whose inside dimensions are 8 inches, 6 inches, *and 4 inches.*

174. Given x a 3-inch line, y a 2-inch line. Build x^3; build y^3; build the rectangular solid x^2y; build the rectangular solid xy^2. Find the volume and facial area of each.

175. Given x a 2-inch line, y a 1-inch line. Build a solid which represents x^3+y^3, one angle of the smaller cube coinciding with one angle of the larger. Find the area of its surfaces.

176. Cut an inch cube out of a corner of a 2-inch cube, and find the volume of the remaining solid and the area of its surfaces.

177. Find the volume and area of surfaces of the solid represented by x^3-y^3, when x=a line of 4 inches and y =a line of 3 inches, y^3 being cut from a corner of x^3.

178. Given x a 3-inch line, y a 2-inch line. Build a solid composed of x^3, $3x^2y$, $3xy^2$, and y^3, making it as nearly as possible in the shape of a cube.

179. Show, by using blocks, that the cube of the sum of two lines equals the cube of the first, plus 3 times the square of the first multiplied by the second, plus 3 times the square of the second multiplied by the first, plus the cube of the second.

180. Find x^3, $3x^2y$, $3xy^2$, and y^3, when x = a 3-inch line and y an inch line and combine them into a cube.

181. Find the area of the surfaces of the irregular solid formed by placing the cube of a line of 4 inches upon the cube of a line of 5 inches, in such a way that the middle point of the lower base of the smaller cube coincides with the middle point of the upper base of the larger, and their edges are parallel.

182. What is the ratio of a 1-inch cube to a 3-inch cube? Of a 2-inch cube to a 5-inch cube?

183. Can you show that the volumes of two cubes are in the ratio of the cubes of their edges? Are they similar solids?

NOTE. — Demonstrative geometry will show you the truth of three closely related principles which will be very useful to you.

PRINCIPLE 56. — *On similar solids homologous lines are proportional to other homologous lines.*

PRINCIPLE 57. — *On similar solids homologous surfaces are proportional to the squares of homologous lines.*

PRINCIPLE 58. — *The volumes of similar solids are proportional to the cubes of homologous lines.*

184. A manufacturer makes stoves of similar patterns but different sizes. Size No. 4 is 3 feet in height; size No. 18 is 6 feet in height. What is the width of the door of the larger stove if the corresponding door of the smaller is 6 inches wide? A brass rod on the larger one is 4 feet 2 inches long. How long is the corresponding rod on the smaller one? The legs of the larger stove raise it 10 inches from the floor. How far is the smaller from the floor?

185. The area of a door of No. 18 is 72 square inches. How many square inches of iron plate are used in making the corresponding door of No. 4? The hearth of No. 4 contains 20 square inches. How many square inches are there in the hearth of No. 18? No. 4 requires 36 square inches of mica. How many square inches of mica will No. 18 require? No. 18 has 80 square inches of nickel

finish. How many square inches of nickel finish are used upon No. 4?

186. The cubic contents of No. 18 are how many times the cubic contents of No. 4? If 75 pounds of iron are used in making No. 4, how many are required for No. 18?

187. Mary's doll, which is 10 inches long, is exactly similar to Anna's doll, which is 30 inches long, and the dolls are dressed alike. If a hand of Mary's doll is 1 centimeter long, what is the length of a hand of Anna's doll? If the belt of Anna's doll is 12 inches long, how long is the belt of Mary's doll? If the hem of the dress of Anna's doll is 33 inches long, how long is the hem of the dress of Mary's doll?

188. If it took half a yard of pink satin to make the dress for Mary's doll, how many yards will it take for the dress of Anna's? If it took one square yard of white plush to make a wrap for Anna's doll, how many square yards are required for a similar garment for Mary's doll?

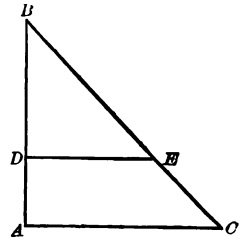
189. If Mary's doll weighs 5 ounces, how many pounds and ounces does Anna's weigh?

190. How many iron balls 2 inches in diameter will equal in weight an iron ball 10 inches in diameter?

191. A vase 8 inches high holds 72 cubic inches. How many cubic inches are there in a vase of the same shape which is 4 inches high?

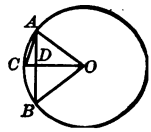
CHAPTER XV.

CUMULATIVE REVIEW NO. 5.



1. BA is perpendicular to AC; DE is parallel to AC. $BA=21$ centimeters; $AC=20$ centimeters; find BC. $BD=14$ centimeters; find DA, EC, and DE. Quote the geometric principles used in solving this problem.

2. The radius of a circle is 10 inches. Find the distance from the center of the circle to the middle point of a chord of 12 inches.



3. The chord AB is 18 inches; radius, 15 inches. Find the length of AC, the chord of half the arc ACB.

SUGGESTION. — Find DO, then DC.

4. Make and solve an original problem whose solution depends upon the fact that a radius perpendicular to a chord bisects it.

5. Express in its lowest terms the fractional part which an arc of 66° is of the circumference.

6. A certain circumference is divided into three arcs. The first is 50° more than the second, and the second 50° more than the third. Find each.

7. How long is each arc given in Ex. 6 if the diameter of the circle is 21 feet?

8. How many degrees are there in an arc of a circle if the remaining arc is 5 times as large? How long is it if the radius is 15 centimeters?

9. Find the perimeter of a sector of 90° in a circle whose radius is $17\frac{1}{2}$ inches.

10. Find the perimeter of a sector of 30° of a circle whose circumference is $16\frac{1}{2}$ inches.

11. Find the perimeter of a segment of 90° in a circle whose radius is 28 centimeters.

12. Find the perimeter of a segment of 60° in a circle whose radius is $38\frac{1}{2}$ inches.

13. Find the difference between the perimeter of a segment of 180° and that of a sector of 180° in a circle whose radius is $52\frac{1}{2}$ millimeters.

14. The complement of a certain angle is 20° more than twice the angle. How many degrees are there in the supplement of the angle?

15. Find the angle which is the G. C. M. of the three angles formed at the center of a circle, the arc of one being 10° greater than one of its adjoining arcs, and 10° less than the other.

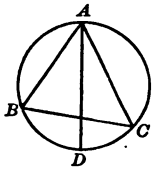
16. How many degrees are there in the arc of the sector which is the G. C. M. of two sectors of the same circle, whose arcs are 80° and 60°?

17. How many degrees are there in the arc of the sector which is the L. C. M. of two sectors of the same circle, the arcs of the sectors being 30° and 40°?

18. When is a segment also a sector?

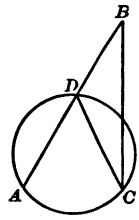
19. Draw a circle, and inscribe in it two right triangles in such a way that they shall not overlap upon each other.

20. Two arcs, one of which is 30° more than 4 times the other, compose a circumference. How many degrees are there in each of the angles which they measure?



21. AD is a diameter. Arc $DC = 50°$. Arc $BD = 80°$. Find each angle of the triangle ABC.

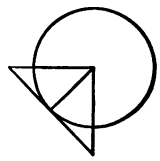
22. How long is the arc intercepted by an inscribed angle of 30° in a circle whose radius is 12 inches?



23 Which is greater, the angle ABC formed by two secants intercepting arc AC, or the inscribed angle ADC, whose sides intercept the same arc? Give reasons.

24. How many times will a wheel 6 feet 5 inches in diameter revolve in traveling 44 feet?

25. Find the distance from the center of a circle whose radius is 13 inches to the middle of a chord 24 inches long.



26. With a radius of 6 inches draw a circle and two radii perpendicular to each other. Bisect by a radius the angle formed by them. Draw a tangent at the extremity of the radius. Prolong the radii which form the angle until they meet the tangent. What is the area of the triangle thus formed?

27. The perimeter of an isosceles triangle is 72 inches. Each of the legs is 6 inches longer than the base. Find each side. Find the altitude and area.

28. The altitude of an inscribed equilateral triangle is what fractional part of the diameter of the circle?

29. The side AB of the scalene triangle ABC is 7 inches longer than the side BC, and BC is 3 inches longer than AC. The perimeter is 25 inches. Find each side.

30. The longest side of a scalene triangle is 5 inches longer than the next in length, and the next in length is twice the length of the shorter side. Find the length of each side if the perimeter is 55 inches.

31. The sum of the legs of an isosceles triangle is 12 meters more than the base, and the perimeter is 48 meters. Find each side. Find the altitude and area.

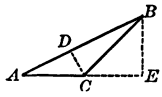
32. Find the angles of an isosceles triangle in which the vertical angle is $\frac{1}{8}$ of the sum of the base angle.

33. A triangle whose base is 12 centimeters and altitude 3 millimeters is what fractional part of one whose base is $16\frac{2}{3}$ centimeters and altitude $4\frac{1}{2}$ millimeters?

34. Draw an equilateral triangle whose side is 3 inches, and find the ratio of its surface to that of an equilateral triangle whose side is one inch.

35. Find the ratio of the surfaces of two equilateral triangles, the ratio of whose sides is 4 : 3.

36. AB is 36 inches. CD, perpendicular to it, is 9 inches; BE, the perpendicular to the line AC, is 20 inches. Find AC.



37. The area of an isosceles triangle is 1452 square feet, its altitude is 44 feet. Find its perimeter.

38. The sum of the equal sides of an isosceles triangle and its altitude equals 49 inches. The difference between one of them and the altitude equals 2 inches. The base is 16 inches. Find the area and perimeter.

QUERIES. — Shall x = the altitude or one of the legs of the triangle? Can you draw an isosceles triangle having its altitude equal to or greater than a leg?

39. Draw the line AB, and a line every point of which is the same distance from A as from B.

Quote the geometric principle which guides you.

40. Find the altitude and area of an isosceles triangle whose base is 40 millimeters, and whose equal sides are each 29 millimeters.

41. Construct equilateral triangles upon each side of a square whose side is 8 inches, and find the area of the starred figure thus formed.

42. The length of a rectangle is 18 millimeters, and its width is $\frac{2}{3}$ of the length. If its width is reduced 3 millimeters, how much must be added to the length that the area may be the same?

43. The width of a rectangle is 12 feet, and its length $1\frac{2}{3}$ times as much. If the length is decreased 4 feet and the width increased 4 feet, what figure is formed? By how much is the original rectangle increased?

44. Draw perpendiculars to each extremity of both diagonals of a square, and prolong them until a surface is inclosed. What figure is thus formed, and what ratio does it bear to the given square?

45. Find the number of square centimeters in the surface of a parallelopiped 4 decimeters long, 3 decime-

ters wide, and 2 decimeters high. How many liters are there in the parallelopiped?

46. Draw a line 5 inches and another 6 inches, and show that the square of the first plus twice the rectangle of their product, plus the square of the second, will give a square 11 inches in its dimensions.

47. Cut out two figures, one the rectangle of the sum and difference of two lines, the other the difference of their squares, and cut and fit one upon the other.

48. Find the area of a regular hexagon, one side of which is 12 centimeters.

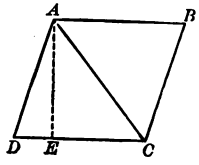
49. The perimeter of a rhomboid is 24 centimeters. The difference between two adjacent sides is 2 centimeters. Find each side.

50. Which is greater, a square whose sides are each 5 inches, or a rhomboid whose base and altitude are each 5 inches? Which has the greater perimeter?

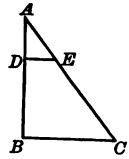
51. Two horizontal parallels 8 inches apart are crossed by two vertical parallels also 8 inches apart. What is the inclosed surface and what is its area? When the vertical parallels are inclined so as to cross at an angle of 60°, the intersections still being 8 inches apart, what is the name of the figure thus formed, and what is its area?

52. The side of a square is 1 foot longer than that of another square, and the sum of their perimeters is 84 feet. Find the sum of their squares.

53. AE the altitude = 10 centimeters; DE = 3 centimeters; EC = 8 centimeters; find the area of rhomboid $ABCD$.

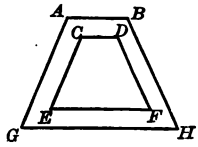


54. $AD=\frac{1}{3}$ of AB. DE is parallel to BC. The perimeter of triangle ABC is 30 inches. Find the perimeter of triangle ADE, and quote the appropriate geometric principles.

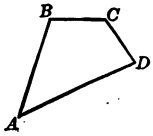


55. A long side of a rhomboid is twice a short side, and the perimeter is 54 inches. Find the perimeter of a similar rhomboid whose short sides are each 6 inches.

56. $ABHG$ is 60 inches. AG and BH are equal, and each is twice AB and 6 inches less than GH. The trapezoids are similar. CD is $4\frac{1}{2}$ inches. Find each side of each trapezoid.



57. The trapezium $ABCD$ has the sides BC and CD equal. AB is 8 inches longer than BC, and AD is 12 inches longer than CD. The perimeter is 56 inches. Find the line which is the greatest common measure of the sides.



58. The perimeters of two similar trapeziums are respectively 27 inches and 36 inches. If the shortest side of the smaller is 6 inches, what is the shortest side of the other?

59. What is the area of the largest circle that can be cut from a piece of paper which is 14 inches square?

60. What is the area of the circular ring bounded by the circumferences of two concentric circles, the diameter of one being 20 inches and of the other 10 inches?

61. A circular plat 40 feet in diameter has a walk around the outside which is 4 feet wide. Find the area of the walk.

62. The circumference of circle A is 3 times that of circle B. A chord of 60° on circle A is 5 centimeters. How long is a chord of 60° on circle B?

63. Draw two parallels and a transversal so that an exterior angle shall be 10° more than its adjacent interior angle, and mark the number of degrees in each of the eight angles formed.

64. One angle of a rhomboid is 70°. Find the ratio of its adjacent exterior angle to the ratio of its consecutive angle.

65. How do you find the number of degrees in the angles of a polygon?

66. How many degrees are there in each angle of a regular polygon of 14 sides?

67. How many degrees are there in each exterior angle of an octagon? Of a pentagon? Of a decagon? Of a dodecagon?

68. What is the ratio of the sum of the exterior angles of a pentagon to the sum of the exterior angles of an octagon?

69. Circumscribe a circle whose radius is 6 inches around a regular hexagon. The perimeter of the hexagon has what ratio to the circumference?

70. A and B start from the same point. A travels north at the rate of 8 miles per hour, B travels west at the rate of six miles per hour. How far apart are they at the end of 7 hours?

71. How many times is a square whose side is 3 inches contained in a square whose side is 5 times as long?

72. How many times is a cube whose edge is 2 inches contained in a cube whose edge is 5 times as long?

73. Name a plane figure that has no angles.

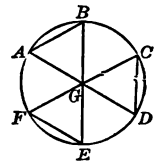
74. Hold a card upright so that one of its edges is on your desk, and keeping it in the same position move it from one side of the desk to the other. The path of the upper edge is what? The path of the card forms what geometric solid?

75. If the card moved from one side of the desk to the other were 8 inches by 5 inches, the desk 15 inches wide, and one of the shorter edges were on the desk, what would be the area of the surface generated by the upper edge? What would be the area of the surfaces of the parallelopiped generated by the card?

76. How many spheres can there be which have a common center?

77. Think of two spheres of different size having a common center. Have they parallel surfaces?

78. *A*, *B*, *C*, *D*, *E*, and *F* are points at equal distances on the circumference of the circle whose radius is $10\frac{1}{2}$ inches, and whose center is *G*. By how much does the sum of the perimeters of the triangles exceed the circumference of the circle? By how much does the sum of the perimeters of the sectors between the triangles exceed the sum of the perimeters of the triangles?



79. Find the edge of a cube the sum of whose faces is 150 square centimeters.

80. Find the volume of a cube the sum of whose edges is 72 decimeters.

81. What line contains all the points in a plane surface which are 30 inches distant from the center of a circle whose diameter is 40 inches. Illustrate.

82. There are two similar monuments, of which the smaller is 3 feet high, the larger 27 feet high. An edge of the base of the smaller is 4 feet; find the corresponding edge of the larger. The area of the top of the larger is 324 square feet; find the area of the top of the smaller. The weight of the smaller is 200 pounds; find the weight of the larger.

83. There are two haystacks of similar shape. One is 10 feet high and contains 4 tons. How many tons are there in the other, which is 20 feet high?

84. Of two cellars of similar shape, the length of the larger, 28 feet, is twice that of the smaller. The width of the larger is 24 feet, and its depth is 10 feet. Find the cubic contents of each. What is the ratio of their contents? Of their surfaces?

85. Of two books of similar shape, one is twice as thick as the other. If the smaller book weighs 3 ounces, how much does the larger one weigh?

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